

WHAT CRITERIA ARE NECESSARY FOR THE SELECTION OF A
TEXTBOOK FOR A SPECIFICALLY DESIGNED SENIOR
MATHEMATICS COURSE?

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by
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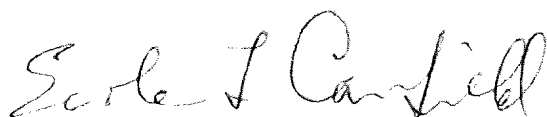
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CHAPTER I

INTRODUCTION

The purpose of the American Secondary School system is to provide an education for all students who desire such an education. Several objectives of the secondary school curriculum, as stated by Romine are "to promote the total and balanced growth and development of boys and girls physically, mentally, emotionally, and socially."¹ The curriculum should be a challenge for all youth in order to measure up to their full capacities.² The process by which the secondary school system attempts to achieve these goals is through a planned curriculum. One part of this curriculum plan is mathematics.

Mathematics has been in existence since the days of early civilisation in and around the Mediterranean Sea.³ It has grown tremendously from these early days of the Babylonians, Egyptians, and Greeks until the present day mathematics. Unfortunately, the mathematics curriculum in the secondary schools has not kept abreast of these changes. Even now there exist many school districts throughout the nation that are

¹Stephen A. Romine, Building the High School Curriculum (New York: Ronald Press Company, 1954), p. 5 of the preface.

²Ibid.

³Howard Eves, An Introduction to the History of Mathematics (New York: Holt, Rinehart and Winston, Inc., 1966), p. 29.

still using mathematics textbooks from the early 1950 editions. Tulock reported that in some elementary schools these conditions are even more severe.¹

Fortunately, the conditions of the past are being removed, and progress is evident in the curriculums. The mathematical knowledge of the past two hundred years was not placed into the secondary mathematics curriculum until recently. The launching of the Russian Sputniks I and II had a great impact on the establishing of the recent mathematical revisions in the American secondary schools. Mathematical curriculum revisions were underway prior to the launchings, but the real impetus came after the launchings when the American public clamored for an improvement in scientific education.² The American people had lost their leadership in the world of science and technology. It was unthinkable for the American people to be second in the realm of scientific knowledge. The mathematics curriculum committees responded to the public demands by recommending course content outlines for grades 7-12 and, only recently, for grades K-6.³

The curriculum revisions for mathematics, "The Queen of the Sciences," resulted in the public's demand for a secondary mathematics

¹Mary K. Tulock, Mathematics Grades 7-12 (Hartford, Connecticut: State Department of Education, 1959), p. 7.

²Stewart Scott Cairns, "Mathematical Education and the Scientific Revolution," The Mathematics Teacher, LIII (February, 1960), 66-74.

³Ibid., pp. 60-70.

curriculum to be in harmony with the proposals set forth by the curriculum committees. Adoption of these proposals was left to the individual school districts as they deemed them necessary.

Today, as never before, high school mathematics teachers are faced with major changes in the mathematics curriculum. The new mathematics outlined and the new methods suggested by groups such as the School Mathematics Study Program (SMSG), the University of Maryland Mathematics Projects, the Commission on Mathematics of the College Entrance Examination Board (CEEb), the University of Illinois Committee on School Mathematics (UICSM), and many other groups have been tried. In general, many of these ideas have been adopted by schools. As a result of these first productions, new mathematics textbooks based on the new programs are being published by book companies. Many of these modern textbooks vary greatly from traditional texts and from each other. These commercially produced textbooks are being reviewed by high school mathematics departments throughout the United States. A particular book, or textbook series, is then selected on the basis of departmental criteria. The curriculum content is established; then a textbook is selected.

This paper will be concerned with the senior mathematics curriculum and the standards used in selecting a new text for the senior mathematics course. The results developed in this paper will be used by the mathematics department of the Davenport Public Schools of Davenport, Iowa.

I. PURPOSE OF THE STUDY

Since a feeling had been growing that a newer series of mathematics textbook should be developed for the senior course at the Davenport Public Schools, the system was faced with this issue in the 1968-1969 school year. However, the mathematics department decided that the curriculum for the senior mathematics course should be revised as well. Consequently, the mathematics department was faced with a dual roll. The first was to decide the course content; the second was that of selecting a textbook to satisfy the course content guidelines.

The primary purpose of the study was to identify and describe the evaluation and selection of a senior mathematics textbook for the senior mathematics course for the Davenport Public Schools. This involved the establishment of a course content guideline as recommended by CEEB, SMSG, NCMT, and the local school district (Davenport, Iowa). Secondly, it involved the establishment of a procedure for the selection of a textbook for the senior mathematics course of the Davenport School District.

This paper will be concerned with the answers to the following questions:

1. What content is recommended for the senior mathematics course?
2. What processes are needed for the selection of a senior mathematics textbook?

II. NEED FOR THE STUDY

Senior mathematics, prior to the curriculum proposals, usually consisted of a semester in solid geometry and a semester of trigonometry.¹ Often, no senior mathematics instruction was offered because of the lack of school personnel or of student interest. However, since plane and solid geometry have been revised and placed in a unified sophomore plane geometry course, a one semester void was created in the senior curriculum. Likewise, the plane trigonometry course was combined with the algebra II course and is now presented at the eleventh grade level. Another semester was left void by the revision of plane trigonometry. As a result, a senior mathematics curriculum was needed and created by the national curriculum committees. From these committees a variety of proposals were suggested and adopted for textbook usage in the twelfth grade program.

There are schools that still offer the traditional plane trigonometry and solid geometry as separate courses. West Central Community School District of Maynard, Iowa, is an example. Most schools,

¹George Greisen Mullinson, Chairman, "Final Report to the Central Association of Science and Mathematics Teachers of Its Committee on the Significance of Mathematics and Science Education," School Science and Mathematics, XXXIV (February, 1954), 119-143.

however, have the senior mathematics course open for the diversified content that is available on the textbook market. The content that has been suggested by the curriculum committees is not as diversified as that indicated by the textbook publishers. The topics that have been recommended by the national curriculum committees include elementary analysis, analytic geometry, probability, statistics, matrices, higher algebra, trigonometry, sequences, and an introduction to calculus.¹ These suggestions have been made by the Commission on Mathematics, the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, the Ball State Program, the College Entrance Examination Board, and the National Council of Teachers of Mathematics.

The textbooks that are available on the market today represent all of these selections in their content. However, the difficulty is that no one curriculum committee has recommended all of these topics. Textbook publishers have, however, incorporated all of these topics. With such a varied and extensive subject area to study, much is covered. However, little depth in knowledge is required in any one area.²

¹From the National Curriculum Committees of the Commission on Mathematics, School Mathematics Study Group, University of Illinois Committee on School Mathematics, Ball State Program, and College Entrance Examination Board.

²Albert E. Meder, Jr., "Sets, Singers, and Salvation," The Mathematics Teacher, LII (October, 1959), 434-438.

The selection of an appropriate textbook for the senior mathematics course, or for any course, is a difficult and technical task for the teaching personnel associated with the school. Every year some textbooks are revised, and the number of textbooks increases. This makes the selection process even more difficult. The superintendent and the curriculum supervisor, as a result, find it nearly impossible to survey and understand the new curricular materials or even to read and evaluate the conflicting analysis made of the material.¹

With the launching of the Sputniks, the mathematics curriculum revisions have been adopted at all grade levels. The first level of education to encounter these numerous revisions was the seventh to ninth grade level of secondary education.² The last area for secondary mathematics curriculum revision was the senior mathematics course. The consensus as to what should be taught is not definite, but most of the national committees did agree that the calculus should be taught at the college level.³ With the recommendations for calculus as a first year college course, what then should be the content of the senior mathematics course? The content, as indicated earlier, is broad indeed.

¹John J. Kinsella, Secondary School Mathematics (New York: The Center for Applied Research in Education, Inc., 1965), foreword of the book.

²Cairns, op. cit., pp. 71-72.

³B. R. Miller, "New Developments in the Teaching of Mathematics," National Association of Secondary School Principals, (April, 1960).

In reviewing several of the textbooks for the senior level, one encounters an even broader area of topics. Such topics are analytic geometry, trigonometry, mathematical structure, logic, methods of proofs, sets, Boolean algebra, functions and relations, mathematical induction, probability, vectors, complex numbers, exponential and logarithmic functions, coordinates, three dimensional geometry, limits, and an introduction to differential and integral calculus.¹ A second textbook for the same grade level offers topics such as number theory, analytic geometry, quadratic functions, trigonometry, exponential and logarithmic functions, mathematical induction, probability, and sets.² The first book contains the material in a very condensed form. The second book has fewer topics, but it has expanded the content and discussion in each area.

The selection of an appropriate senior mathematics textbook is further complicated if the school offers the traditional plane trigonometry course as a one-semester course in the senior year. Such was the case at West Central Community School and the Davenport Public School District. Such an approach to the senior course makes the

¹Abraham M. Glickman and Harry D. Ruderman, Fundamentals for Advanced Mathematics (New York: Holt, Rinehart and Winston, Inc., 1964), pp. v-ix.

²Theodore Herberg and James D. Bristol, Elementary Mathematical Analysis (Boston: D. C. Heath and Company, 1962), pp. v-x.

development of selection criterion even more difficult because of a lack of one-semester textbooks.

Textbook selection, until recently, was done by the local school board and superintendent.¹ Today most teachers are asked to participate in the development of course content and in the selection of appropriate textbooks for use in their specific subject areas. Teachers have assumed the responsibility for textbook selection willingly. These are the individuals who use the textbooks, and they should be responsible for their selection.² However, teachers who have been involved in the selection and evaluation of textbooks have realized that the techniques for the proper evaluation are extremely limited. According to Reeve, textbook evaluation was based on personal opinion and previous experience with the content of a certain textbook.³ The factors considered in the selection must be more specific in order to obtain the best textbooks available for school use. The importance of the selection of a textbook can be best expressed by the viewpoint taken by a committee of educators as indicated

¹Malcolm E. Mellott, "What to Look for In Choosing a Textbook," NEA Journal, XXXIV (March, 1955), 158-159.

²Pershing Vartanian, "Criteria and Techniques for Textbook Evaluation," The Social Studies, LIII (April, 1962), 123-127.

³William David Reeve, "How to Choose a Textbook," School Science and Mathematics, (November, 1955), 601-609.

in the introduction to the Thirteenth Yearbook of the National Society for the Study of Education:

The significant position of the textbook in the program of American Education is so generally recognized that the Society seems to be fully justified in sponsoring a yearbook on the theme "The Textbook." It is the textbook that in thousands of classrooms determines the content of the instruction as well as the teaching procedures. This statement may not be in accord with the usual theory but it is supported by the facts reported by supervisors and state inspectors of schools. In view, therefore, of the important place of the textbook in our educational practice, the preparation and the selection of textbooks is a problem of major importance.¹

With the need of a new textbook to be selected and a new course content to be established for the Davenport School District, the mathematics coordinator established a committee for this need. The committee decided early in its investigation of curriculum and textbook selection that the senior mathematics course was in large measure dependent upon the material from a textbook; thus, the importance of the book could not be overemphasized. The committee also realized that some teachers deviate from the pattern of the book and try to individualize their teaching from the textbook. However, the textbook still provides the framework for the course and sets the pattern for the type of instruction which is given.

¹National Society for the Study of Education, Thirteenth Yearbook, Part II, "The Textbook in American Education," Public School Publishing Company, Bloomington, Illinois, 1931, p. 1.

In view of these facts, it would seem that one effective way to improve the content in the senior mathematics course would be to place superior textbooks in the hands of the students and teachers. Textbooks that presented adequate content and suggested methods and procedures that enabled a teacher to make the senior mathematics meaningful to the student were considered for selection. While authors and publishers of most current senior mathematics textbooks claim to meet these requirements, it was soon discovered that there is still apparently room for improvement.

As a result of the need for a set of goals to be established for a full year senior mathematics course, the content of the course was determined first. Then criteria for the selection of a senior mathematics textbook were established.

III. METHOD OF PROCEDURE

After it had been decided that the senior mathematics course was to have a new textbook in the Davenport Public Schools, a committee was established for this purpose. The selection of the textbook was made by a committee of five senior high mathematics teachers. After spending several months reviewing currently published material for the senior mathematics curriculum, a curriculum was established. Upon establishing a curriculum, a review of the currently published texts using the modern approach was made.

The criteria used for the selection of the senior mathematics textbooks were determined after a general procedure was developed for

evaluating a given textbook.

This paper will establish the curriculum for the senior mathematics course for the Davenport Public Schools. Having established a curriculum, criteria will be developed for the selection of an appropriate textbook. A complete description of the sampling procedure is included in Chapter III.

The method of procedure used for the development of the set of standards required four parts:

1. Surveying the literature on the selection and usage of a textbook.
2. Collecting and reviewing the national curriculum committee proposals for mathematical content.
3. Selecting a set of criteria from the first two parts and the development of a rating chart involving the established criteria.
4. Comparing two senior mathematics textbooks for a year's course.

The sources of the literature read during part one of the report were from The Education Index, Readers Guide to Periodical Literature, NEA Journal, Journal of Educational Research, The Mathematics Teacher, The Arithmetic Teacher, School Science and Mathematics, Bulletin of the National Association of Secondary School Principals, and The Midland Schools. The library card catalogue was investigated also. Because of the lack of information available for textbook selection, much of the information was obtained from the writings in the various education periodicals. As the investigation proceeded, the criteria from the

national committees and textbook selection guidelines were collected and tabulated.

Selected textbook companies were contacted for suggestions relative to the subject of the report. The information from the sources mentioned was analyzed to develop a set of criteria to be used during the third part of the investigation. In order to make this information understandable, a rating scale was established and demonstrated in the final part of the study.

IV. LIMITATIONS

The investigation for selecting a senior mathematics textbook revealed insufficient guidelines to make an adequate selection. No mention was given of the selection of a senior mathematics textbook by the curriculum committees. The only criteria available were general guidelines with no specifics.

The purpose of the study was to develop a senior mathematics course and a useful tool for the evaluation of a textbook for the course for the Davenport Public Schools. If the textbook under consideration was not readily readable or if the quality of material and format were not desirable, then the textbook was not to be considered. If the textbook should fail for these reasons, then the evaluation procedure would not be needed.

V. DEFINITIONS OF TERMS

The following terms are defined to help clarify this study:

Senior mathematics. Senior mathematics refers to the mathematics that is taught during the senior year to college bound students. Its content consists of trigonometry, statistics, probability, theory of equations, exponential and logarithmic functions, and other topics. In addition, these students have had three years study of mathematics including algebra, plane and solid geometry, Algebra II, trigonometry, and analytic geometry.

Modern mathematics. Modern mathematics refers to the trend toward a study and classification of the patterns developed in mathematics in which insight is an important part of the formal deductive reasoning process.¹ Understanding is stressed rather than recall in the learning process.

Traditional mathematics. Traditional mathematics refers to the manipulative development of number symbols which implies an emphasis on the deductive reasoning and recall rather than the intuitive approach to the understanding of mathematical concepts.²

Criteria. The principles for selection or statements of quality level against which a book is measured or compared.

¹ Meder, op. cit., p. 408.

² Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, "Secondary Mathematics Curriculum," The Mathematics Teacher, LII (May, 1959), 369-407.

Concepts. Concept was the term used to refer to a generalized idea or class of things.

Problems. Problems refer to the word or verbal type of situation in which the material is written in detail. An example is: "the length of a rectangle is twice as great as the width. The perimeter of the rectangle is 43 inches. Find the length, width, and area."

Exercises. Exercises refers to the drill material such as equations to solve and various operations to perform.

Axiomatic approach. An axiomatic approach has the following items of importance for the development of new ideas or processes: (1) undefined terms, (2) defined terms, (3) axioms (or statements accepted without proof as a basis for argument), and (4) theorems.

Because of their familiarity and simplicity, the following abbreviations will be used in this report.

UICSM. The University of Illinois Committee on School Mathematics.

CEEB. The Commission on Mathematics of the College Entrance Examination Board.

SMSG. The School Mathematics Study Group.

NEA. The National Education Association of the United States.

NCTM. The National Council of Teachers of Mathematics.

VII. ORGANIZATION OF THE FIELD REPORT

This report is organized into five chapters. A brief description of each of these chapters is as follows:

Chapter I is primarily concerned with the statement of the problem, its importance, and the general procedure for the investigation.

Chapter II contains a comprehensive review of the literature which appeared to have some important significance in the development of the set of criteria for the report. Chapter II also lists some of the factors that brought about curriculum changes and recommendations for changes. It also contains reviews of various articles which contain reports on some experimental studies, a list of writers in the field, and summaries of textbook selection guidelines.

Chapters III and IV contain the development of the material and a demonstration of its use. Chapter III contains a discussion of the criteria, references to the sources, a description of the situation, the school, and the textbooks used. This is followed by a summary of the data, and a guideline is established for use. Chapter IV demonstrates the usage of the guidelines in selecting a textbook. The textbook will be selected with reference to the course content established by the committee which followed recommendations of the various national curriculum groups for the content of the senior mathematics course.

Chapter V summarizes the study and contains the conclusions drawn from the study.

An appendix and a bibliography conclude the study.

CHAPTER II

CURRENT LITERATURE AND RESEARCH

This chapter is devoted to an examination of the literature pertaining to the problems of the content of the senior mathematics course and the selection of a textbook for the senior mathematics course. An examination of the current literature has revealed that there is an abundance of opinions on the question of a mathematics curriculum. There is, however, a limited amount of literature concerning the selection of a textbook. There are relatively few good research studies available to assist one in the selection of a senior mathematics curriculum and the selection of a mathematics textbook for the senior course. Consequently, this chapter will include a cross section of these opinions as well as the research information available.

This chapter will be divided into the following nine basic parts:

1. Introduction.
2. Causes for curriculum change.
3. Some modern curriculum programs.
4. Mathematics curriculum committees.
5. Some opinions on modern curriculum programs.
6. Some opinions on the importance of the textbook to the curriculum.
7. Some causes for teacher involvement in the textbook selection.

8. Some opinions on the procedure for textbook selection.

9. Summary.

The sources of information for the review of literature contained in this paper include: published educational periodicals, complete dissertations available on the Drake University campus, curriculum committees, experimental curriculum projects, score sheets for textbook selection, and published reports dealing with secondary school education and mathematics.

I. INTRODUCTION

Prior to World War II, the mathematics curriculum of the secondary school was relatively stable. With the declaration of the Second World War there was a general awakening to the inadequacy of mathematics training when high school graduates did not score well on military induction tests. High schools began to require more mathematics for graduation, and there was a general increased emphasis on competence in mathematics.¹ The inadequacies became more apparent when the Russians launched their first Sputnik in 1957. The consequences of the War and the launchings have resulted in tremendous advancements in the scientific knowledge available for usage by society. This has had a profound effect on current mathematics curricula. Some of the recent writers indicate that a considerable change has occurred in the field of mathematics. In

¹David William Reeve, Mathematics for Secondary Schools (New York: Holt, Rinehart and Winston, Inc., 1960), pp. 515-517.

a booklet published by the National Council of Teachers of Mathematics,

Price stated:

The changes in mathematics in progress at the present time are so extensive, so far reaching in their implications, and so profound that they can be described as a revolution.¹

In addition, mathematics was not deemed important by the public and there was a general over-all loss of interest in mathematics in the schools. This situation was described by Reeves as follows:

It is most interesting and also instructive to note that it took a world war to make the American people generally realize that basic mathematical instruction in the schools of this country was woefully deficient. Many of us knew a long time ago that such conditions existed. Leaders in the educational field had previously pointed out the shortcomings, but the deficiencies thus disclosed did not at the time seem to be so tragic because the public was not made aware of the possible dire results of such a sad state of affairs, as then existed.²

The knowledge of mathematics did not seem necessary to protect our freedom.³ However, after the World War II changes were started though by the time Sputnik arrived, our mathematical deficiencies were still acute and noticeable.

In addition to the changing of the curriculum content in mathematics, the general teaching pattern of mathematics has changed. Current research in the behavioral science is influencing the pattern

¹G. Bailey Price, "Progress in Mathematics and Its Implication for the School," The Revolution in School Mathematics, National Council of Teachers of Mathematics, A report of regional orientation conference in Mathematics (Washington: 1961), p. 1.

²Reeve, op. cit., p. 512.

³Ibid., pp. 515-517.

of education. The teaching of children how to think rather than what to think is stressed so that they can face the problems of their age.¹

II. CAUSES FOR CURRICULUM CHANGES

In the 1950's major changes were initiated in the mathematics curriculum. These changes in the mathematics curriculum were caused by many factors. Some of these factors will be discussed. They include: recognition of the problem of curriculum deficiencies by mathematics educators, social pressures, employment opportunities, new uses for mathematics, new learning theory, and new application of mathematics in the technical fields.

The recognition of the problem of curriculum deficiencies by Mathematics Educators. The inadequacy of mathematics to fulfill the needs of our society was recognized by many mathematics educators. Johnson pointed out this need and recommended more and better mathematics, especially for the better mathematics students.² Included in his report was a checklist to evaluate a present mathematics curriculum.

Fehr also was aware of the situation; the following is an outline suggested for changes.

¹Robert S. Fleming (ed.), Curriculum for Today's Boys and Girls (Columbus: Charles E. Merrill Books, Inc., 1963), p. 21.

²Donovan A. Johnson, "The Spotlight Is on Mathematics," Minnesota Journal of Education, (January, 1957), 14-15.

1. Look at the existing mathematics programs.
2. Decide what should be taught so that capable students can go into areas involving rigorous mathematics.
3. Decide how mathematics can be presented so as to produce an interest; hence, a larger number of graduates.
4. Make recommendations for modification of existing programs.¹

Much of the literature in the mathematics journals was related to the problem of curriculum change. Although the experts in mathematics education are not entirely in agreement, most advocated some type of change in the existing mathematics programs. These changes will be discussed in more detail in a later section of the paper.

Social pressures. In 1957 the Russians launched their first Sputnik, and the public became aware of the fact that another country had made a technical advance beyond the United States. As a result, there was a change in public concern for mathematics and science education. This concern was reflected by the amount of money set aside by the Federal Government in the National Defense Education Act of 1958. Over one billion dollars was to be spent on education.² Broad areas, such as student loans, research and experimentation, science and mathematics instruction, and laboratory equipment, were included in the early

¹Howard Fehr, "New Thinking in Mathematics Education," The Mathematics Teacher, (October, 1960), 424-429.

²Milton W. Beckman, "NDEA Support for Improved Mathematics Instruction," The Mathematics Teacher, (October, 1960), pp. 445-447.

provisions of this act.¹

Various curriculum committees were formed and financed by universities, private organizations, and the federal government. The Carnegie Corporation financially supported the Maryland Project, the University of Illinois Committee on School Mathematics, and the Commission on Mathematics of the College Entrance Examination Board. The Ford Foundation financially supported the Commission of Admission to College with advanced standings and the Mathematical Association's Committee on the Undergraduate Program. The National Science Foundation supported the School Mathematics Study Group and the Mathematical Association's Committee on the Undergraduate Program.²

All of the above financial sources have helped to implement the various curriculum changes in mathematics. These changes have been brought about because society recognized the need for a change in this area.

Employment opportunities. As a result of the involvement in the war, a general deficiency in mathematics was evident. Parrish pointed out the shortage of scientific, professional, and technical manpower who had a good mathematics background.³ He concluded that this manpower shortage could be remedied by broadening our mathematics objectives. It might then be possible to keep a larger portion of the student population

¹Ibid.

²Ibid.

³Clyde E. Parrish, "Junior High School Mathematics and the Manpower Shortage," The Mathematics Teacher, (December, 1956), 614.

interested in mathematics.¹

New uses for mathematics. The uses of mathematics have been greatly increased. This has been an extremely important factor in the change of the mathematics curriculum. As indicated by Adler:

Mathematics has spilled over from physics into the other physical sciences, chemistry, and geology. It has invaded the life sciences, biology, and psychology, and has expanded into the social sciences too. There is no area of science today that can avoid using mathematical methods.²

In addition, there are new uses for both the traditional and the modern mathematics. Research in the areas of geophysics, the earth's magnetic field, mechanical engineering, and radar communications has brought about new applications of traditional mathematics. Mathematics is now being used in the social sciences to handle the large masses of data that have been and are being accumulated. Industry is using mathematics for quality control and sampling theory. Economics is using mathematics to study the economy. New applications of mathematics are increasing as our society becomes more complex.³ As was stated by Gibl:

Every citizen in our society needs mathematics and must understand it if he is to communicate effectively with those about him

¹Ibid., pp. 611-616.

²Irving Adler, "The Changes Taking Place in Mathematics," The Mathematics Teacher, (October, 1962), 450.

³College Entrance Examination Board, Commission of Mathematics, Program for College Preparatory Mathematics (New York: 1959), pp. 3-5.

and if he is to comprehend the operation of government and the material he reads in the newspaper.¹

New learning theory. From 1920-1940 the education of the mathematics teacher was influenced by the connectionistic theory. The connectionist theory states that a complex learning task can be successfully completed by learning in isolation and sequence each of the constituent tasks. This theory was applied to the teaching of mathematics and resulted in the show-do-method of mathematical instruction.² In the connectionist theory, the teacher would demonstrate a process; then the students would proceed and practice the operation. However, by 1950, the field theory began to be dominant in the education of the mathematics teachers. The field theory advocates the development of concepts, meanings, understandings, generalizations, and problem solving. The textbooks written after 1954 reveal the influence of the field theory of instruction.³

The preceding topics discussed indicate some of the factors causing a mathematics curriculum change. However, these factors have also had an influence on the development of the mathematics curriculum.

III. SOME MODERN CURRICULUM PROGRAMS

Since 1950, many curriculum projects have produced experimental mathematics materials. These experimental programs have been tried in

¹E. Glenadine Gibl, "Basic Objectives of the New Mathematics," Education Digest, XXXI (December, 1965), 45-47.

²Kinsella, op. cit., pp. 63-69. ³Ibid., pp. 71-73.

the classroom. The following experimental programs will be discussed as they pertained to the senior mathematics curriculum.

1. University of Illinois Committee on School Mathematics.
2. School Mathematics Study Group.
3. Ball State Experimental Program.
4. College Entrance Examination Board.

The University of Illinois Committee on School Mathematics. The University of Illinois Committee on School Mathematics was appointed in December, 1951, by the Deans of Education and Engineering and the head of the mathematics department. The committee's major objectives were to improve the mathematics competence of the beginning engineering students to the point where they would be capable of studying calculus during the freshman year of college. It was decided that to be realistic any improvement would have to include classroom tested material at the high school level.¹

McCoy pointed out that the writing staff had these five major points:

1. A consistent exposition of high school mathematics is possible.
2. High school students are very interested in ideas.
3. Manipulative skills, though necessary, should be primarily used to emphasize basic concepts.
4. The language used in textbooks or heard from the teacher should be as unambiguous as possible.

¹Max Beberman, "Improving High School Mathematics Teaching," Educational Leadership, (December, 1959), 162-169.

5. The materials should be organized in such a fashion that students would have abundant experience in discovering generalizations.¹

The University of Illinois Committee on School Mathematics felt that by the implementations of these five principles, the student would develop an understanding of mathematics and an enthusiasm for mathematics.

In 1958, the first mathematics teaching units for grades nine through twelve established by the University of Illinois Committee were available. The teaching units were distributed only to teachers who had special training for the materials. This training was obtained by enrolling in the National Science Foundation sponsored summer institutes.²

The teaching units, accompanied by teachers' commentaries, were available by 1961. The material prepared by the University of Illinois Committee of School Mathematics had two outstanding characteristics:

1. Teaching units were so designed that by reading carefully and by doing the exercises conscientiously the student would discover principles of mathematics.
2. Explanations by the teachers are reduced to a minimum.³

¹Eleanor McCoy, "The Secondary Mathematics Program," The National Association of Secondary School Principals, (May, 1959), 12-18.

²National Council of Teachers of Mathematics, An Analysis of New Mathematics Programs, 1963, p. 58.

³Kinsella, op. cit., p. 28.

School Mathematics Study Group. In 1958, a small organizing committee of educators and university mathematicians was appointed by the president of the American Mathematical Society to organize a School Mathematics Study Group. The organizing committee appointed Edward G. Begle as its director. The committee consisted of college and university mathematicians, high school mathematics teachers, experts in education, and representatives of technology who were appointed to assist the director, Mr. Begle. The National Science Foundation provided financial support to the School Mathematics Study Group.¹

The School Mathematics Study Group pointed out the following:

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and understanding the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that the mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.²

The School Mathematics Study Group stated the following objectives:

1. We need an improved curriculum which will offer students not only basic mathematical skills but also a deeper understanding of the basic concepts and the structure of mathematics.

¹Edward G. Begle, The School Mathematics Study Group, The National Association of Secondary School Principals, (May, 1959), p. 26.

²School Mathematics Study Group, Newsletter No. 6 (New Haven: Yale University Press, March, 1961), pp. 3-4.

2. Mathematics programs must attract and train more of those students who are capable of studying mathematics with profit.
3. All help must be provided for the teachers who are preparing themselves to teach these challenging and interesting courses.¹

The School Mathematics Study Group has produced materials from grades four through twelve and teachers' manuals to accompany the materials. Units at all grade levels, four through twelve, have been tested in the classroom and have been revised.² These units have been published and are available for use. There have been some attempts to evaluate the School Mathematics Study Group. An evaluation of the material for grade twelve will be discussed later in this chapter.

Ball State Experimental Program. The Ball State Experimental Program was started in 1955 at Ball State Teachers College. Text materials were prepared by Charles Brumfiel, Robert Eicholz, and Merrill Shanks. The project started with the development of a tenth grade geometry text. The project has now completed a beginning and an advanced algebra text. Content for the senior mathematics course is not definite, but ideas contained in this program for grade eleven were constructed with those stressed for grade twelve by the other study groups under discussion. The content for senior mathematics course stressed topics from the School Mathematics Study Group. The major objective of the Ball

¹Ibid.

²An Analysis of New Mathematics Programs, op. cit., p. 33.

State Experimental Program was to develop the student's ability to make proofs from clearly stated assumptions.¹

IV. MATHEMATICS CURRICULUM COMMITTEES

In addition to the actual production of mathematics curriculum materials, the various mathematics curriculum committees have urged curriculum change. Two of the most recognized committees are the Secondary School Curriculum of the National Council of Teachers of Mathematics and the Commission on Mathematics of the College Entrance Examination Board.

The Commission on Mathematics of the College Entrance Examination Board. The Commission on Mathematics of the College Entrance Examination Board published recommendations for college preparatory mathematics from grades nine through twelve in 1958.

The major proposals of the Commission on Mathematics of the College Entrance Examination Board are outlined in the following nine points for the high school mathematics curriculum:

1. Strong preparation, both in concepts and in skills, for college mathematics at the level of calculus and analytic geometry.
2. Understanding of the nature and roll of deductive reasoning - in algebra as well as geometry.

¹Charles Brumfiel, Robert Eicholz and Merrill Shanks, "The Ball State Experimental Program," The Mathematics Teacher, (February, 1960), 75-84.

3. Appreciation of mathematical structure ("patterns") - for example, properties of natural, rational, real and complex numbers.
4. Judicious use of unifying ideas - sets, variables, functions and relations.
5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some coordinate geometry, and essentials of solid geometry and space perception.
7. Introduction in grade eleven of fundamental trigonometry - centered on coordinates, vectors, and complex numbers.
8. Emphasis in grade twelve on elementary functions (polynomials, exponential and circular).
9. Recommendations of additional alternative units for grade twelve: either introductory probability with statistical applications or an introduction to modern algebra.¹

The Commission on Mathematics called for relatively minor changes in content but recommended changes in instructional techniques for the presentation of the material.

Secondary School Curriculum Committee. The Secondary School Curriculum Committee was organized by members of the National Council of Teachers of Mathematics in 1958. The committee was headed by Frank B. Allen. The Secondary School Curriculum Committee studied the following areas:

1. The place of mathematics in our changing society.
2. The aims of mathematics education.

¹Program for College Preparatory Mathematics, op. cit., pp. 33-34.

3. The nature of mathematics taught in grades seven through twelve.
4. How geometry should be introduced and taught.
5. Content and organization of junior high mathematics.
6. Foreign mathematics for pupils of ages twelve to eighteen.
7. Adjustment of the mathematics programs to pupils of exceptional ability.
8. Aids in teaching.
9. The organization of the mathematics program.
10. The administration of the mathematics program.¹

The Secondary School Curriculum Committee published a report in The Mathematics Teacher in May, 1959. The following summarizes the report of the Secondary School Curriculum Committee:

Objectives: The objectives for the mathematics courses must be made in relation to the situation in which we now live.

The Secondary School Curriculum Committee stated:

1. The statement of objectives should indicate both the desired behavior and the type of situation in which it is to occur.
2. The objectives should be stated in terms of desired pupil behavior rather than teacher behavior.
3. The objectives should be formulated at a level of specificity such that for each objective it is possible to infer some learning activities appropriate for helping pupils achieve it, and also such that it is possible to devise means of

¹Frank B. Allen, "A Report of the Secondary School Committee," The Mathematics Teacher, LI (February, 1958), 571-575.

evaluating the achievement, but not to a greater degree of specificity.¹

For the content of the mathematics curriculum, the Secondary School Curriculum Committee made the following recommendations:

1. Precise language.
2. The concept of sets can be used to an advantage at all grade levels.
3. Equations and inequalities should be taught using solution sets, truth of an equation, the idea of identity, and conditional equations.
4. Emphasis on mathematical models requiring that properties be stressed at all mathematics course levels.
5. The use of reasons for statements at all grade levels.²

Implementation of the mathematics curriculum: The Secondary School Curriculum Committee recommends the following for implementing a mathematics curriculum:

1. Provide for individual differences by: (a) having a four year sequence of mathematics for all students in grades nine through twelve, (b) having a two year requirement of mathematics for all high school graduates, (c) having special classes for low-ability students with a maximum classroom enrollment of twenty students, (d) locating student deficiencies and providing remedial instruction, and (e) moving pupils into a more advanced level of instruction if they show a promise for mathematical achievement.³

¹Secondary School Curriculum Committee, "The Secondary Mathematics Curriculum," The Mathematics Teacher, (May, 1959), 395.

²Ibid., pp. 397-400.

³Secondary School Curriculum Committee, op. cit., p. 408.

2. Provide an organized program by: (a) having an appropriate minimum program for every educable individual whether a slow learner, average pupil, or mathematically gifted, and (b) providing a sufficient program to challenge the interest and the ability of the most able student.¹
3. Provide proper administration for the mathematics curriculum by close cooperation between the teacher and the administrator.²
4. Providing qualified teachers is the most important single factor in any mathematics program. The Secondary School Curriculum Committee recommends that a mathematics teacher: (a) should have competence in all areas of mathematics, (b) should have completed a five-year program, (c) should have at least eighteen semester hours of teacher education, and (d) should keep currently informed in mathematics education by reading journals, such as, The Mathematics Teacher, and by participating in mathematics institutes.³

V. VARIED OPINIONS ON MODERN MATHEMATICS PROGRAMS

Prominent mathematicians and mathematics educators have debated the merits of the mathematics curriculum reform. Price reported that the implications of the revolution in mathematics for the teaching of mathematics are quite clear. That is, the present mathematics curriculum and its presentation need a revision to satisfy the needs of our time.⁴

Kline contended that there was no need for a drastic change in the mathematics content. Kline stated the following:

¹Ibid., p. 411. ²Ibid., p. 413.

³Ibid., p. 414.

⁴Price, op. cit., p. 1.

Genuine improvement of the curriculum does call for drastic revision, though not of course by resorting to modern mathematics and not by merely reshuffling the traditional mathematics.¹

In addition, Kline mentioned the following statement:

The trouble is not that we are teaching outmoded mathematics, except for one or more topics. Rather, the trouble lies in the way in which we approach the material we teach.²

Jones indicated that the modern mathematics curriculum does not mean total abandonment of the old mathematics. It does not mean a reckless introduction of new mathematics or a new text. It does, however, need a new text that would be an aid in teaching.³ Jones also indicated that modern mathematics required an examination of time allotment, and it required continued study and modifications by the mathematics teacher.⁴

At the Third Annual Symposium on Engineering Mathematics, college and high school mathematics teachers reacted to questions developed from a report on college preparatory mathematics by the Commission on Mathematics of the College Entrance Examination Board. The reactions of the college and high school mathematics teachers are summarized as follows:

¹Morris Kline, "The Ancients Versus the Moderns," The Mathematics Teacher, (October, 1958), 425.

²Ibid.

³Phillip S. Jones, "The Mathematics Teacher Dilemma," Education Digest (May, 1959), 34-37.

⁴Ibid.

1. Eighty-seven per cent favored extensive revision of the present high school curriculum.
2. Seventy-nine per cent thought every potential college student should complete three years of high school mathematics.
3. Ninety-seven per cent agreed that calculus is a college subject. Most high schools with ordinary mathematics staffs should not attempt to teach calculus.
4. Eighty-seven per cent favored increasing mathematical rigor in ninth grade algebra.
5. Ninety-seven per cent thought more emphasis on deductive reasoning in the ninth grade was desirable.
6. Ninety-one per cent agreed coordinates should be introduced in the tenth grade after six weeks.
7. Ninety-two per cent thought solid geometry should be eliminated as a separate course and should be integrated with the tenth grade geometry.
8. Ninety per cent agreed the twelfth grade course should include one semester of elementary functions.
9. Sixty-nine per cent agreed that rudimentary trigonometry should be taught in the ninth grade.¹

Genise, working with the University of Maryland Mathematics Project and the School Mathematics Study Group, reported that the materials of these two curriculum committees have been accepted enthusiastically by teachers, students, and parents.²

The Ball State Program had the following observations made by Brumfiel:

¹ Arthur W. Leissa and Robert C. Fisher, "A Survey of Teachers Opinions of a Revised Mathematics," The Mathematics Teacher (February, 1960), 113-117.

² Roland L. Genise, "The Impact of the Maryland and Yale Programs," The Arithmetic Teacher, (February, 1960), 66-70.

1. Teachers with a strong background in mathematics are enthusiastic about the Ball State Program.
2. Teachers with a weak background in mathematics have mixed reactions.
3. Capable students are enthusiastic: and brilliant students perform magnificently.
4. Weak students cannot follow patterns, but seem to perform as well as when studying traditional materials.
5. Average students show more variation: some perform well and others lose interest.
6. There has been no increase in dropouts due to the increase in rigor.¹

Criticism has been leveled at the modern mathematics program for the placement of too much rigor in the high school mathematics curriculum. Rappaport questioned modern programs for their placement of sophisticated content prior to the student's ability to comprehend the concepts.²

VI. OPINIONS OF THE IMPORTANCE OF A TEXTBOOK TO THE CURRICULUM

After a curriculum has been established for a given course, the next step is the selection of the material for this planned curriculum. The material most frequently selected is the textbook. The question then arises as to what textbook will be best suited to the given situation.

¹Brumfiel, Eicholz, and Shanks, op. cit., pp. 75-84.

²David Rappaport, "Does 'Modern Math' Ignore Learning Theory?", Phi Delta Kappan, XLIV (June, 1963), 446.

Obviously, there is no one textbook that is superior in all respects to every other textbook. Consideration of the local needs will be a large factor in the selection of the textbook which is to be used. The choice must not be made at random. No one can merely leaf through a book for an hour or two and completely judge the book as to its merit or weakness. Wise judgment will be based on objective data which show the quality and quantity of the subject matter contained in the textbook.

The importance of the textbook as an aid to the teacher in his teaching a given course is a valuable tool. The importance of the textbook was stated by Hall-Quest:

The skill in handling the textbook is just as important as skill in handling the tools in manual training or household arts.¹

The textbook should serve as a guide for the teaching of mathematics at a given grade level. However, the importance of the textbook cannot be underestimated. According to Margolis:

The importance of the textbook is becoming ever more important since too many teachers rely on textbooks and the accompanying manuals to shape the course work, day to day lesson plans, and the assigning of homework. The textbook is the leader not the teacher.²

The importance of the textbook to the curriculum is obvious. Sterning noted that the textbook would be a necessity since there

¹Alfred Lawrence Hall-Quest, The Textbook: How to Use and Judge It (New York: The Macmillan Company, 1920), (Preface).

²Richard Margolis, "The Well-Tempered Textbook," The Education Digest, (December, 1965), 24-27.

existed so few scholars in each subject area available for classroom instruction.¹ Consequently, the textbook acts as an "assistant teacher in print" and should be the responsibility of the teacher for its selection if the textbook is of this importance to the curriculum.²

At the center of the present day educational scene in America is the textbook. The textbooks, rightly or not, take a dominant place in the typical school from kindergarten to college. The only other educational devices that are used as much are the black-board and writing material. Cronbach further indicated that the textbook is in universal use.³

VII. CAUSES FOR TEACHER INVOLVEMENT IN THE TEXTBOOK SELECTION

Since the teacher is responsible for the instruction in the material and the textbook is the main source available for the information that is to be dispersed to the pupils, it then should be the responsibility of the teacher and the principal to select an appropriate textbook. Horitz indicated that the textbook will continue to be the most important item among the instruction material and that the teacher

¹John Sterning, "How To Select New Textbooks," Nations Schools, LXXVII (March, 1969).

²Mellott, op. cit., pp. 158-159.

³Lee J. Cronbach, Text Materials in Modern Education (Urbana, Illinois: University of Illinois Press, 1955), p. 173.

and the principal are the most concerned with the books and their uses.¹ The selection of a textbook should be made by this team of the teacher and the principal. Further, the teacher is in the closest contact with the student and his textbook, and the responsibilities for the selection of the instructional material should lie with the teacher.²

Additional research indicated that the selection of a textbook should be made by the teacher and principal, and the recommendations of these two should be given to the superintendent. The superintendent should acknowledge and accept the recommendations of this team. Strager noted that the student should be thought of first in the selection of a textbook and that the superintendent should accept the recommendations of the principal and teacher in their selection of a given textbook.³

VIII. OPINIONS OF THE PROCEDURE FOR TEXTBOOK SELECTION

The previous two units in this chapter (Opinions on the Importance of the Textbook to the Curriculum and Causes for Teacher Involvement in

¹Harry E. Horitz, "Teachers Can Help Evaluate and Select Textbooks," Elementary School Journal, (February, 1956), 250-254.

²
Ibid., p. 254.

³George D. Strager, Problems In Educational Administration (New York: Columbia University, 1925), pp. 567-570.

the Textbook Selection) were placed in this field report to indicate the importance of the selection of an appropriate textbook by the teacher for a given curriculum. How the textbook is to be selected is the next consideration.

The procedure for the actual selection of a textbook is quite difficult. The usual procedure is to follow certain guidelines in selecting a textbook. Many articles were read and studied in an attempt to find a criteria for the selection of a senior mathematics textbook. Most of these articles provided a general guideline, but few specific procedures were given for the actual selection of a senior mathematics textbook for the Davenport Public Schools.

One of the books investigated was titled: The Teacher In Curriculum Making, and it stated that there were three aspects to the problem of the selection of a textbook:

1. The broadest was the potentiality.
2. Educational value is of paramount consideration.
3. Material must contribute to the outcomes sought.¹

In a somewhat dated book, Textbook Selection, the major criteria for the selection of a textbook were given by these five items:

1. The factor of interest.

¹Joseph Leese, Kenneth Frasure, and Mauritz Johnson, Jr., The Teacher In Curriculum Making (New York: Harper and Brothers, Publishers, 1961), pp. 232-236.

2. The factor of comprehension.
3. The permanent methods of study involved in the text.
4. The permanent value of the text.
5. The mechanical construction of the text.¹

The article further stated that the school administration should presumably subscribe to the following four fundamental practices to sound textbook selection:

1. The factors of (a) interest, (b) comprehension, (c) permanent methods of study involved in the text, and (d) the mechanical construction of the textbook.
2. Objective evidence should be given preference over subjective opinion wherever objective evidence can be reasonably obtained.
3. Whenever subjective opinion is resorted to it should not be the result of any one person but of several competent individuals.
4. Wherever subjective opinion has to be substituted for objective measurement, especial care should be taken to get the best opinions possible.²

Letters of inquiry were sent to many publishing companies with the intention of receiving guidelines for the selection of a senior mathematics textbook. The following companies were contacted: Scott, Foresman and Company; Houghton Mifflin Company; Holt, Rinehart and Winston, Inc.; D. C. Heath and Company; D. Van Nostrand Company, Inc.;

¹R. H. Frazen and F. B. Knight, Textbook Selection (Baltimore, Maryland: Warwick and York, Inc., 1922), p. 11.

²Ibid., pp. 13-14.

Allyn and Bacon, Inc.; and Silver Burdett Company. Replies from each of the companies indicated that the general procedure for the selection of a senior mathematics textbook was not an easy task and that the guidelines sought were quite limited. Allyn and Bacon, Inc., in a letter stated:

Mathematics education and, in particular, present trends in modern mathematics, have been largely influenced by the invaluable experimental work of various research study groups. Although the recommendations of these study groups are incorporated in textbooks and provide guidance for curriculum planners, it is the local school officials who determine the scope and content of their own mathematics program. Since different educational goals require different text materials, course syllabi vary among individual states and among local cities and towns. Consequently, in the selection and publication of our mathematics textbooks, there is no one particular syllabus that we follow.¹

In a reply from Scott, Foresman and Company, they stated the following:

An outline or a guideline that an individual should use for the selection of a textbook for the senior mathematics course is not available.²

D. Van Nostrand Company, Inc., D. C. Heath and Company, Holt, Rinehart and Winston, Inc., Houghton Mifflin Company, and Silver Burdett Company returned no material for textbook selection. Their replies were that the selection of textbook materials is a task of the local teacher.

¹Lorraine F. Stigar, Personal Letter from Allyn and Bacon, Inc., July, 1968.

²Ruth Draper, Personal Letter from the Customer Service Representative of Scott, Foresman and Company, August, 1968.

The material that they did send was a course outline of their series of mathematics textbooks. This was not the information desired, however, for this field report.

The Department of Public Instruction for the State of Iowa was contacted, and their reply was a small report which had the following items:

1. How to use the mathematics textbook?
2. Why a textbook is needed in mathematics?
3. What are the contributions of good mathematics textbook?
4. How is the mathematics textbook properly used?
5. What are the dangers involved in textbook teaching?
6. What qualities does a good mathematics textbook have?
7. How is a mathematics textbook evaluated?¹

Item number seven (How is a mathematics textbook evaluated?), from the State Department of Public Instruction, listed the following items as being important:

1. Content
2. Organization
3. Aids to learning
4. Physical aspects
5. General comments by the evaluator.²

¹Department of Public Instruction, State of Iowa, A five page report on mathematics textbooks, 1967, pp. 1-4.

²Ibid., p. 4.

The Des Moines Public Schools, the Allamakee Public Schools of Waukon, Iowa, and the Cedar Rapids Public Schools were contacted in regard to their procedure for the selection of a senior mathematics textbook. The Cedar Rapids Public Schools replied with a one-page score sheet which had the following general items to be considered for evaluating mathematics textbooks:

1. Structure:

- A. Does the presentation assist the student in understanding the structure of this particular area of mathematics?
- B. As an extension of a topic is made, does the development show clearly how the extension is related to the structure under consideration?

2. Rigor:

Rigor in a text refers to the nature of the development of the arguments and the kind of justification that is used in proof. Few presentations are entirely rigorous or completely without rigor. The level of rigor in a text may have much to do with the future understanding of the subject by its reader.

- A. Is the development of the topic made on appropriate levels of rigor?
- B. Does the author attempt to cultivate and capitalize on the reader's intuitive understanding, while pointing out the dangers and limitations of dependence on intuition?¹

The Allamakee School District of Waukon, Iowa, replied with criteria for the selection of English and Elementary Health textbooks.

¹Cedar Rapids Public Schools, Evaluating Mathematics Textbooks, 1968, p. 1.

The guidelines that were considered of importance for the senior mathematics textbook selection procedure were as follows:

1. Authorship within the subject area.
2. Inclusiveness of program whereby:
 - A. Does the series present information and concept development in (mathematics)?
 - B. Does the series over-emphasize any one phase of (mathematics)?
 - C. Does the series have needless repetition?
3. Content: (Material was for health only.)
4. Approach
 - A. Does the series take a positive approach to (mathematics)?
 - B. Does the series take examples from all walks of life, rather than just the ideal situation?
 - C. Are questions and situations, as well as examples, provided at frequent intervals throughout each chapter to motivate class discussions in regard to the material just read?
 - D. Do functional and useful activities appear at the end of each chapter?
 - E. Does the series correlate with other curriculum areas rather than just within its area of instruction?
 - F. Are there numerous sub-headings to aid the student in what he is reading?
5. Teaching aids
 - A. Are the complete students' content included in the Teacher's Edition?
 - B. Are references made for aids in additional instructional material?
 - C. Is the teacher given information for:

1. Points to emphasize.
 2. Materials which help to preview the unit.
6. Physical features
- A. Attractiveness.
 - B. Illustrations.
 - C. Diagrams.
 - D. Binding and cover.
 - E. Printing and spacing.¹

The Des Moines Public Schools had the following items of importance to be considered for the selection of a textbook: interest, comprehension, scope and permanent value of the content, method, and mechanical elements. These five areas were to have a value of 1000 points in the numerical rating score card.²

In addition to the five areas listed for textbook selection, the Des Moines Public Schools had a criteria for elementary textbook selection. The elementary score sheet had the following areas to be considered and their point value:

1. Authorship and Publisher. (50 points)
2. Approach or Method. (300 points)
3. Content. (450 points)

¹Allamakee Public Schools, Waukon, Iowa, Criteria for Evaluating Health and English texts, 1968, pp. 1-4.

²Des Moines Public Schools, Des Moines, Iowa, Manual for Text-book Selection Committees, 1968, p. 3.

4. Teachability Factors. (130 points)
5. Format. (70 points)¹

A third score card was also sent by the Des Moines Public Schools. This score card, for the evaluation of mathematics textbooks, was for the applied mathematics texts.

After reviewing these three score cards or score sheets, the writer was convinced that their usage for the selection of a senior mathematics textbook would not be adequate. The general guidelines would be of some value as will be indicated in Chapter Three. The criteria for the selection of a senior mathematics textbook were not available from the Des Moines Public Schools. The writer felt that the score cards provided by the Des Moines Public Schools would be helpful but were not completely adequate. This fact was further acknowledged by the Mathematics Department Head, Mr. A. Wilson Goodwin, who stated:

We are not satisfied with the construction and content of our score cards. These should be built from a well-constructed set of behavioral objectives in mathematics. These we don't have.²

Our own school district, the Davenport Public Schools, had a one-page score sheet for the selection of mathematics textbooks. The one-page score sheet had the following format:

1. An interesting review of fundamentals.

¹Des Moines Public Schools, Des Moines, Iowa, "Criteria for the Evaluation of Elementary Mathematics Texts," pp. 1-4.

²A. Wilson Goodwin, Supervisor Department of Mathematics, Des Moines Public Schools, A personal letter, March 10, 1969.

2. The book must be up-to-date. (Modern concepts are what we need in a new textbook, not just added facts.)
3. All illustrations should be meaningful and attractive.
4. It should be generally attractive and up-to-date in appearance.
5. It should present a good balance of subject matter in the field it is intended to treat.
6. Its content should be organized so as to lead logically from one area of study to another.
7. Its study helps should be helpful and useful to the learning process. (Avoid "busy work.")
8. Its use should be flexible enough to meet the needs of all students for which it is chosen. (Including readability.)
9. Its author(s) should have followed goals in writing it, which successfully coincide with ours.¹

These guidelines were too general in nature and not specific enough for the selection of a senior mathematics textbook. The committee decided that an interesting review of fundamentals could embrace a wide range of mathematical content and could take a full semester simply in reviewing. Consequently, Item I was not specific enough. For the rest of the items to be considered, in textbook selection, the writer decided that the items listed would be used as reference as he established a new score sheet for the selection of a senior mathematics textbook.

The last review of literature for the selection of a senior mathematics textbook was made from the writers of the mathematics

¹Davenport Public Schools, Davenport, Iowa, "What to Look for In New Mathematics Textbook," 1968, p. 1.

curriculum or leading mathematics educators. The item considered first was the readability of the material for the grade level it was intended. The readability of the material would be an important factor for the selection of a textbook. As was stated by Smith and Heddens:

It is apparent that the readability levels of the experimental mathematics materials studied are considerably higher than their assigned grade levels. A need for revision of the materials to place the readability on a more appropriate level would seem to be evident. If teachers are to use the present materials effectively, they will find it necessary to provide instruction in reading mathematics materials, and will in addition find it necessary to provide more than the usual amount of oral explanation.¹

In addition to the readability of the material, the precision of what is written is an important item to be considered for the selection of a mathematics textbook. As stated by Schminke:

Writers, both educators and mathematicians, have attempted to explain elements of basic concepts in various articles published in periodicals, pamphlets, and texts for consumption largely by persons engaged in classroom teaching and mathematics education. . . . Therefore, in order that we may utilize the printed material with children or reach a new understanding of a concept being discussed, it is imperative that the thought, the vocabulary, and the illustrative computation presented in professional literature be accurate, precise, and consistent. It is time for precision.²

¹ Kenneth J. Smith and James W. Heddens, "The Readability of Experimental Mathematics Material," The Arithmetic Teacher, II (October, 1964), 393-394.

² G. W. Schminke, "A Time for Precision," The Arithmetic Teacher, II (October, 1964), 395.

Schminke further stated:

It would therefore seem appropriate to urge that writing and publication in the field of mathematics education, by anyone and for whatever level, should place a premium on consistency, clarity, and precision. Certainly, it is not unrealistic to expect the published literature of a discipline to possess the very precision and clarity it seeks, if it is to be of any real use to conscientious teachers.¹

The importance of the discovery approach in the instruction of mathematics has been greatly stressed. The textbook publishers stress that their books use the discovery approach in their presentation of the material. However, the importance of the discovery approach for the selection of a mathematics textbook can be overemphasized. As stated by Adler:

A fourth retarding theory is that the child must discover by himself, without teacher guidance, everything that he must learn. Now it is important to give children opportunities to make discoveries on their own level, but it is also important to recognize the limitations of the discovery method of teaching. We cannot expect the unaided student to discover in a few years what mankind as a whole took thousands of years to discover. Discovery in the classroom must be guided discovery, and often it should be cooperative discovery.²

The selection of a textbook involves several areas of importance. This was indicated by the committee on Aids for Evaluators of Textbooks. They said that the choice of a textbook was dependent upon the particular situation in which it was to be used. The areas to be considered of

¹Ibid., p. 401.

²Irving Adler, "The Cambridge Conference Report: Blueprint or Fantasy?", The Mathematics Teacher, LIX (March, 1966), 215-216.

importance were the philosophy of the school system and the abilities of the pupils; the mathematical competence of the instructional staff must also be considered. The formulation of a mathematics program for a particular school system was the responsibility of the teachers and the administrators within the system. The committee further recommended that the opinions of the teachers who use the textbooks should be taken into consideration.¹

The Committee on Aids for Evaluators of Textbooks stated that the following items concerning mathematics content should be evaluated when choosing a mathematics textbook:

1. Structure
2. Rigor
3. Vocabulary
4. Definitions and undefined terms.
5. Correctness
6. Theorems and proofs
7. Generalizations
8. Ordering
9. Texts, exercises, and reviews
10. Illustrative examples
11. Teachability examples

¹National Council of Teachers of Mathematics, "Aids for Evaluators of Mathematics Textbook," The Mathematics Teacher, LVIII (May, 1965), 467.

12. Optional topics.¹

These ideas were similar to some of the ideas of the reviewed score cards and score sheets from the schools contacted. The items mentioned by the Committee on Aids for Evaluators of Textbooks were the most detailed encountered by the writer. However, this evaluation procedure was for the selection of any mathematics textbook.

The importance for the selection of a mathematics textbook, (or for that matter any textbook), was indicated by Bradley:

Modern textbooks have some good features, that's why they were published. The challenge, if not the truth, in selecting such textbooks is to pick the one that is best suited for the particular children who will use it.²

There has been much written in the mathematics journals in the 1950's and 1960's relating to the problem of curriculum change. Committees have made suggestions for curriculum revisions. Curriculum committees have written materials for classroom use and this material is now in textbooks throughout the country.

Since the curriculum content for a senior mathematics course tended to vary from one author to another and from one mathematics committee to another, the curriculum for the senior mathematics course

¹Ibid., pp. 468-472.

²R. C. Bradley, "These Guides Show How to Select Math Texts for Elementary Schools," Nations Schools, LXXV (July, 1965), 33.

at the Davenport Public Schools had no definite plan to follow. Consequently, the senior mathematics course was developed by using ideas from several sources. Among the sources used were the curriculum reports and the writing of mathematics educators. This curriculum was developed as a result of the review of the literature available.

The criteria for the selection of a senior mathematics textbook were non-existent for the Davenport Public Schools. The review of literature yielded no criteria for the selection of a senior mathematics textbook. However, the review of literature and the review of score cards and score sheets yielded several and varied ideas for the development of a score card for the selection of a senior mathematics textbook. The score card was developed with the use of those available, and the ideas set forth were the result of the review of literature.

The results of the curriculum for the senior mathematics course and the criteria for the selection of a senior mathematics textbook for this course are set forth in Chapter Three.

CHAPTER III

BACKGROUND OF THE STUDY

As was stated in Chapter One, the problem of the present study was to develop a course content for the senior mathematics course in the Davenport Public Schools and to develop a selection procedure for a textbook for the senior mathematics course. This chapter includes a discussion of the background of the study. The topics included in this chapter are: (1) the school and community, (2) the students for whom the senior mathematics course is intended, (3) the curriculum pertaining to the senior mathematics course, (4) the senior mathematics course content, (5) the development of a procedure for the selection of a senior mathematics textbook, and (6) the materials used for the development of the senior mathematics course and the development of ideas used for the textbook selection procedures.

I. THE SCHOOL AND COMMUNITY

The Davenport Public Schools consist of a school population of 24,601 pupils. There are twenty-four elementary schools, an oral deaf center, five junior high schools with a sixth under construction, and two senior high schools. The two senior high schools, Central and West, have an enrollment of 2,300 and 2,200 students, respectively.¹

¹Davenport Community School District, Directory, 1968-1969, p. 148.

The mathematics curriculum prior to the curriculum revision of this report was operating from guidelines established in 1954. Solid geometry and trigonometry were the courses offered for the two semester senior mathematics course. Solid geometry was using a textbook dated 1954, and the trigonometry course was using a textbook dated 1934 and revised in 1952.

II. STUDENTS FOR WHOM THE SENIOR MATHEMATICS COURSE WAS INTENDED

Prior to the establishment of this committee for mathematics curriculum revision in 1968, the first semester of the senior mathematics course for the advanced mathematics student was restricted to solid geometry for students who had only a course of plane geometry. However, the second semester of the senior year consisted of trigonometry for students who had plane geometry, Algebra I, II, and III, and the solid geometry course.

The committee felt that each year many students were enrolled in solid geometry with a too limited background for the course. Consequently, the senior mathematics course was revised and set up under the following definition as stated by the Commission on Mathematics of the College Entrance Examination Board:

The advanced mathematics. . . it is assumed that

the students taking the course are anticipating further college work in mathematics or science.¹

The senior mathematics was further defined as a course for seniors who have had Algebra I, geometry, Algebra II and advanced algebra, analytic geometry, and trigonometry prior to their enrollment in the stated course. The reason the course was limited to seniors only was stated by the Davenport Public School Administration:

No student shall be permitted to take more than one mathematics or science course in any one semester. It is the purpose of our school system to provide a broad education to each and every pupil. Consequently, if a student is permitted to enroll for more than one course within an academic semester, a danger exists that we will have provided only a partial education for the student. We cannot turn all students into mathematicians, engineers, or scientists.²

III. THE CURRICULUM PERTAINING TO THE SENIOR

MATHEMATICS COURSE

In order to carry out the revisions for the senior mathematics course, it was necessary to relate the senior course to the total mathematics curriculum of the Davenport Public Schools. The curriculum needed to be current and have available research and material for effective comparison, evaluation, and ultimately, the final selection for the content of the senior mathematics course.

¹Commission on Mathematics of the College Entrance Examination Board, "Modernizing the Mathematics Curriculum" (A directive for superintendents of schools and members of school boards), 1958, p. 8.

²Davenport Public Schools, An Administrative Directive to the Mathematics Curriculum, 1967, p. 2.

The task for developing this specific area of the mathematics curriculum, the senior mathematics course, was given to four people. The supervision was done by the Director of Mathematics Curriculum and Instruction, Mr. Harlan Goldsmith. Assisting Mr. Goldsmith were Mr. Nagy, Mrs. Hiatt, and the writer of this report. The selection for the committee was determined prior to the 1968-1969 school year.

Prior to establishing the senior mathematics course content, the committee felt that a general statement should be formulated or found from the literature of the various mathematics curriculum reports. The general guideline deemed most beneficial for the mathematics curriculum (and specifically for the senior mathematics course) was indicated by the following statement:

Mathematics is a cumulative and a continuously expanding subject in both its organization and its application. With every new topic the teacher is confronted with four basic instructional problems: (1) helping the students acquire initial understandings of new concepts and relationships, (2) helping them to strengthen and deepen these well beyond the point of mere "threshold" understanding, (3) helping them maintain understandings and skills already attained, and (4) helping them build the background for significant transfer of these skills and understandings to their physical, social, and intellectual environment. These four phases of teaching should be interwoven as far as possible into a unified instructional program, but their implications are essentially distinct and supplemental rather than identical. The teaching of new material necessarily draws upon the already established background as a frame of reference and to this extent serves as a means of maintenance, but such maintenance is relatively incidental to the mastery of the new material and must be so regarded. Adequate maintenance and maximum transfer, especially of skills, cannot be assumed by incidental contacts

but requires an instructional program designed especially for their attainment.¹

It was further deemed essential that objectives be established for the mathematics curriculum for the Senior High School level of the Davenport Public Schools. The general objectives established for the mathematics curriculum were as follows:

1. To establish an appreciation for the development of mathematics and its importance in creative activity.
2. To show the interrelation among the various branches of mathematics.
3. To call attention to the character of mathematics in the theoretical and practical development of science and technology.
4. To encourage each student to study and to use the mathematical language as a tool and to gain satisfaction through personal achievement.
5. To develop the basic mathematical skills and concepts that every citizen should know.
6. To provide intellectual challenges for each student while effecting a transition of the logical structure of mathematics to life situations and activities.
7. To develop mathematical competence in each pupil to the level of his capacities and encourage his own resourcefulness.
8. To develop attitudes toward mathematics that lead to appreciation, confidence, initiative, respect, and independence.²

¹Charles H. Butler and Lynwood F. Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill, 1965), p. 133.

²Davenport Public Schools, "General Objectives of Mathematics Education," 1968, p. 1.

Placement of a pupil in the appropriate level of instruction for the pupil's capabilities is a challenge of the educational system. Consequently, the curriculum committee deemed it essential to proceed with the development of a proper guidance procedure for placement of the pupils in the Davenport Schools. For proper placement, one must realize the background, the abilities, the interest, and the individual needs of the students. In addition, the students are advised of the nature of the courses offered. When it is considered necessary they are cautioned against a program which could result in eventual failure. In guiding students toward mathematics, the following criteria are used:

1. Desires of the pupil and parents.
2. Past achievements in mathematics.
3. Recommendations of former mathematics teachers.
4. Scores in aptitude tests.
5. Scores in standardized tests.
6. Any other items which are of help.¹

With the framework established for the instructional procedure for the course selection, the mathematics curriculum outline was established for the Davenport Public Schools. The basic mathematics curriculum outline was established so that every student is required to have at least one year of mathematics in grades ten through twelve.

¹Ibid., p. 2.

The following is a brief outline of the mathematics courses offered and the grades for which each course is intended.

1. Mathematics I

This basic course is designed for those students who have had difficulty in arithmetic. Electives in grades ten, eleven, twelve.

2. Algebra I

Elective in grades nine, ten, eleven, or twelve.

3. Mathematics II

(Algebra is a prerequisite)--This course, designed for students not enrolling in geometry. Elective in grades ten, eleven, or twelve.

4. Geometry

Algebra is a prerequisite. This course is an integrated course in Plane and Solid Geometry. Elective in grades ten, eleven, or twelve.

5. Intermediate algebra

The first semester course. Algebra and Plane Geometry are prerequisites. Elective in grades eleven and twelve or in the tenth grade for certain selected students.

6. Advanced algebra

A second semester course. Intermediate algebra is a prerequisite. Elective in grades eleven and twelve or in the tenth grade for certain selected students.

7. Trigonometry

A first semester course offered in the first semester only. Advanced algebra is a prerequisite. Elective in grades eleven and twelve. Eleventh grade for selected students.

8. Analytical Geometry

A one semester course offered in the second semester only. Trigonometry is a prerequisite. Elective in grade twelve or in the eleventh grade for certain selected students.

9. Advanced mathematics (Senior mathematics)

Analytical geometry is a prerequisite. Offered only to seniors and is elective.

These are the mathematics courses offered in the Davenport Public Schools. No attempt was made to establish the content for each of the courses listed for this report. The content outline for each course is available, but its placement in this report would require another thirty-five pages besides the justification for the selection of the content for each course. The course outline with which this report is concerned is the senior mathematics course. Senior mathematics was the replacement for the one semester solid geometry and the one semester trigonometry courses formerly offered in the senior year.

Since the senior mathematics course was an addition to the existing mathematics curriculum, the newly formed addition to the curriculum was developed according to these three principles:

1. The content must be based on the existing curriculum and consist of modification and improvement of the present program.
2. The content should emphasize the study of mathematics creatively rather than in terms of rules and tricks.
3. The content must be adequately far reaching so as to represent the modified curriculum in terms of present and future needs.¹

IV. SENIOR MATHEMATICS COURSE CONTENT

The content for the senior mathematics course was developed in a pattern which would not duplicate the previous curriculum content.

¹Commission on Mathematics of the College Entrance Examination Board, op. cit., pp. 5-7.

Recommendations for the senior mathematics course were studied from the following curriculum committees: the Commission on Mathematics of the College Entrance Examination Board, the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, and the Ball State Teachers College Experimental Program.

The Commission on Mathematics of the College Entrance Examination Board felt that no ideal sequence of topics existed for every school situation. They did, however, suggest the following topics:

Elementary functions, first semester course.

I. Sets and combinations

A review and extension of concept of set; symbolism; subsets; null set; union; intersection; complement; venn diagrams; solution sets; and graphs.

II. Functions and relations

- (a) Sets of ordered pairs; Cartesian set ($u \times v$)
- (b) Functions: definition (set of ordered pairs), domain, graphical test; methods of determining functions (table, graph, formula, rule); range of function values; notation $F(x)$, (F denotes the function); (optional) functions as mappings.
- (c) Relations: definition, domain, range; function as a special kind of relation; graphs (equations, inequalities, including absolute value-e.g., $|x| + |y| = 1$).
- (d) Inverse relations and functions; graphical interpretations and tests.

III. Polynomial functions

- (a) Brief review of linear and quadratic functions.
- (b) The general polynomial: definition, degree; remainder theorem, factor theorem, (optional) synthetic division; graphs of ax^n (n a small integer).
- (c) Slope of graph at point (x_0, y_0) :

$$m(x_0) = \lim_{x \rightarrow x_0} \frac{F(x) - F(x_0)}{x - x_0}$$

intuitive, numerical, and graphical discussion of this limit;
tangent line
 $y - y_0 = m(x - x_0)$;
(optional) interpretation as instantaneous rate.

- (d) Slope function m ; application to curve tracing (turning points); other easy applications (maxima and minima, optional) rates.
- (e) Polynomial equations: definition; graphical location of roots; graphical approximation of irrational roots; fundamental theorem of algebra (sufficiency of complex number system); number of roots, factorization; conjugate complex roots.

IV. Exponential functions

- (a) Review of definition, properties, and graph of a^x over the rational domain (similar to grade II).
- (b) Extension to base a ($a > 0$, $a \neq 1$); properties of function carry over.
- (c) Graphs; $y = \log_a x$ ($a = 2, 3, 10$); compare logarithmic and exponential graphs (reflection in $y = x$).

V. Circular functions

- (a) Radian measure; $a = r\theta$; $A = \frac{1}{2}r^2\theta$; related problems.
- (b) Definitions of $\sin x$ and $\cos x$ for real numbers x (wrapping x - axis around a unit circle); domain and range; $\tan x = \sin x / \cos x$; relation to trigonometry of angles.
- (c) Graphs: $\sin x$, $\cos x$, $\tan x$, periodicity; $a \sin (bt + c)$, amplitude, period, phase; (optional) graphs such as that of $\sin 2t + 3 \cos t$ by addition of ordinates.
- (d) Inverse \sin and inverse tangent: graphs, domain (restricted to obtain function), range.
- (f) (Optional) Solution of trigonometric equations; evaluation of expressions such as $\sin (\arctan 3/4)$.
- (g) (Optional) Power series for e^x , $\sin x$, $\cos x$ discussed informally ("non-terminating polynomials"); Euler's formula:

$$e^{ix} = \cos x + i \sin x$$

Second Semester Alternatives

The course on Elementary Functions outlined was considered as a minimum for a four-year program as proposed by the CEEB. With the above outline for a first semester course completed, the following alternatives were suggested by the Commission for a second semester:

¹Commission on Mathematics of the College Entrance Examination Board, "Program for College Preparatory Mathematics," 1959, pp. 42-43.

Alternative 1:

Introductory Probability with Statistical Applications

- I. The nature of probability and statistics.
- II. Organization and presentation of data - the frequency of distribution.
 - (a) The dot frequency diagram and cumulative graph for ungrouped measurements.
 - (b) The frequency histogram and cumulative polygon for grouped measurements.
 - (c) Preview of statistical inference based on the frequency distribution.
- III. Summarizing a set of measurements - the mean and standard deviation.
 - (a) The mean of a set of measurements and properties of the mean.
 - (b) The standard deviation and other measures of variability of measurements.
 - (c) Computation of the mean and standard deviation for ungrouped and for grouped measurements.
 - (d) Chebyshev's theorem for a frequency distribution.
- IV. Intuitive approach to probability.
 - (a) Single repeated experiments with coins, dice, and cards; the notion of sample space and event; mutually exclusive and complementary events.
 - (b) The notion of probability of an event; independent events; conditional probability.
 - (c) Random numbers and their use in simple experiments.
- V. Formal approach to probability.
 - (a) More formal treatment of sample spaces; events, compound events; probability; probability of compound events from probabilities of simpler events; conditional probability; independence.
 - (b) Mathematical expectation.
- VI. The law of chance for repeated trials - the binomial distribution.
 - (a) Case of 2 and 3 trials, extension to n trials.
 - (b) Binomial probability tables and their use.

VII. Applications of binomial distribution.

- (a) Simple cases of acceptance sampling.
- (b) Simple examples of industrial acceptance sampling plans.
- (c) Testing from the results of many trials, a hypothetical value of the probability of success in a single trial.

VIII. Using samples for estimation - sampling from a finite population.

- (a) Using samples to make estimates of population means.
- (b) An illustration of sampling without replacement.
- (c) Preview of the relations between populations and samples drawn without replacement.
- (d) The means of the frequency distribution of sample means.
- (e) The variance of the frequency distribution of sample means.
- (f) Sampling with replacement.
- (g) Mean and variance of sums of measurements drawn with replacement.
- (h) Random sampling.
- (i) Mean and variance of the binomial probability distribution.
- (j) The law of large numbers.¹

Alternative 2:

Introduction to Modern Algebra (Fields and Groups)

I. Fields

- (a) The system of rational numbers.
 - 1. Recall rational numbers and their algebraic properties.
 - 2. List of properties of addition, subtraction, multiplication (i.e., axioms for a ring).
 - 3. Careful deduction of theorems (the "rules" of elementary algebra).
 - 4. Axiom on existence of inverse; definition of quotient (fraction).
 - 5. Deduction of properties of fractions.
- (b) A similar system with two elements (even and odd under + and ·).
- (c) The abstract field (Keeping "+" and "·" as operations, but observing abstract nature of consequences of the axioms).

¹CEEB, op. cit., pp. 44-45.

- (d) Other fields.
 1. Integers under $+$ and \cdot modulo 3 (show why modulo 4 gives a field, by more advanced properties of prime numbers).
 2. Real numbers under ordinary $+$ and \cdot .
 3. Complex numbers, defined as ordered pairs of real numbers, with axioms for a field proved.

II. Ordered Fields

- (a) Recall properties of relation for reals.
- (b) Axioms for ordered fields, taking " $<$ " as primitive.
- (c) Deduction of rules for manipulating inequalities (including relation \leq).
- (d) Examples of ordered fields: rational numbers; real numbers.
- (e) Definition of least upper bound, greatest lower bound.
- (f) Real numbers described as an ordered field in which every bounded set has a least upper bound.

III. Abelian (i.e., commutative) groups

- (a) Introduced by examples of groups of numbers:
 1. Sets of numbers closed under multiplication and division ($[1, -1]$, $[1, i, -1, -i]$, all rationals, all powers of 2, etc.).
 2. Sets of numbers closed under addition and subtraction (all integers; all multiples of $1/3$; all complex integers, integers modulo 3).
- (b) Axioms for a multiplicative abelian group.
 1. Restatement of axioms in additive notation.
 2. Proof that inverse is unique, show $b^{-1}a$ is solution of equation $bx=a$, explanation of what this means in additive case.
- (c) Study of examples:
 1. Any set of numbers closed under subtraction is a group.
 2. Any set of non-zero numbers closed under product and reciprocal is a group.
 3. Integers modulo m under addition.
 4. Complex m th roots of unity.
- (d) Isomorphism (optional).
 1. Examples of isomorphic groups.
 - i. $[1, -1]$ under multiplication, with $(0, 1), (\text{mod } 2)$ under addition.
 - ii. All integers under addition with all powers of 2 under multiplication.
 2. Definition.
 3. Cyclic groups of order m (3c and d) above.

IV. Transformation, composition.

- (a) Basic ideas.
 - 1. Geometric examples of transformation groups.
 - 2. Define a transformation $T: A \rightarrow A^1$ as a function.
 - 3. Composite $S \cdot T$ of two transformations: $(S \cdot T)(x) = S(Tx)$.
 - 4. Identity transformation I .
- (b) Examples of transformation groups.
 - 1. All rotations of a regular polygon.
 - 2. All permutations of two things.
 - 3. All translations of a line, of a plane.
 - 4. All motions of a triangle or square (not commutative!).
- (c) Transformation groups.
 - 1. Definition of inverse transformation, $S \cdot T = I = T \cdot S$.
 - 2. Proof that inverse is unique of present.
 - 3. Define transformation groups (set of transformations of a set A , closed under composition and inverse).

V. Groups (not necessarily commutative).

- (a) Axioms (assuming two-sided identity and inverse).
- (b) Prove; every transformation group is a group.
- (c) Inverses: uniqueness, inverse of product, ab^{-1} and $b^{-1}a$ as solutions of equations.
- (d) Division algorithm $m = qn + r$ for integers (in preparation for exponents to follow).
- (e) Exponents and order of an element.
 - 1. Definition in abstract group of a^n , a^{-1} , a^{-n} .
 - 2. Cyclic subgroup generated by an element a .
 - 3. Order of element, using division algorithm.
- (f) Examples: numerous examples of transformation groups, especially those drawn from geometry.¹

The preceding two-one semester courses were given in complete detail. The third alternative given by the CEEB was a course of selected topics. These topics are flexible and could be used with the courses offered as Alternative I or Alternative II. The selected topics were as follows:

¹Ibid., pp. 45-46.

Selected Topics

- I. Additional work on sets, functions, and relations.
- II. Mathematical induction.
- III. An extension of permutations, combinations, and the Binomial Theorem.
- IV. Probability.
- V. Inequalities and absolute values, solution sets and graphs.
- VI. Graphing factorable polynomials and rational functions.
- VII. An emphasis on graphical interpretations of systems of equations.
- VIII. Three dimensional coordinate geometry. Topics of planes, lines, and spheres.¹

These course outlines, as provided by the CEEB, provided the greatest amount of information possible for the content of the senior mathematics course. However, the content of the Selected Topics as indicated by the CEEB for the senior mathematics course was in duplication of the content offered in advanced algebra, trigonometry, and analytical geometry. Namely; (a) graphing factorable polynomials and rational functions, (b) permutations and combinations, (c) inequalities and absolute values, and (d) graphical interpretation of systems of equations and solution sets. Consequently, the course outlined by the CEEB had to be studied in detail and certain items deleted. These items of deletion will be discussed later in this chapter.

The second curriculum committee course outline reviewed was that of the University of Illinois Committee on School Mathematics. The

¹Ibid., p. 47.

course outline, as indicated by UICSM, is as follows:

I. Circular functions

- (a) Winding functions
- (b) Periodicity
- (c) Evenness and oddness
- (d) Monotoneity
- (e) "Analytical trigonometry" rather than "triangle solving"
- (f) Inverse circular functions

II. Deductive Theories

- (a) Abstraction of postulates from a model.
- (b) Deduction of theorems from these postulates without reference to a model.
- (c) Reinterpretation of the theory to yield information about other models.¹

This content outline was quite limited, however. Further research yielded that UICSM was advocating the senior mathematics course outline issued by D. C. Heath and Company in their textbook, Elementary Mathematical Analysis by Herberg-Bristol.² The content from this book was as follows:

- 1. Set notation and its use.
- 2. Numbers, functions and relations.
- 3. More about relations and functions.
- 4. The linear function.
- 5. Limits and continuity.
- 6. The derivative.
- 7. The antiderivatives.
- 8. Quadratic functions and relations.
- 9. Polynomial functions.

¹Studies In Mathematics Education, Scott, Foresman and Company, 1960, pp. 32-34.

²Max Beberman, UICSM Mathematics Project, March, 1968. (A Newsletter.).

10. Polynomial equations.
11. Exponential and logarithmic functions.
12. Circular functions I.
13. Circular functions II.
14. Solution of triangles.
15. Complex numbers and vectors.
16. Introducing matrix algebra.
17. Permutations, combinations, probability.
18. Mathematical induction.
19. Mathematical structures.¹

The third curriculum study that was considered for the establishment of a senior mathematics course was that of the school Mathematics Study Group. Their course outline was as follows:

Course title: Elementary Functions (a one semester course).

- I. Functions.
- II. Polynomial functions.
- III. Tangents to graphs of polynomial functions.
- IV. Exponential and logarithmic functions.
- V. Circular functions.²

The second semester course as outlined by the SMSG curriculum was as follows:

Course title: Introduction to Matrix Algebra (a one semester course offered in the second half of the senior year).

- I. Matrix Operations.
- II. The algebra of 2×2 matrices.
- III. Matrices and linear systems.

¹Herberg and Bristol, op. cit., pp. vii-xii.

²School Mathematics Study Group, Newsletter No. 7, April, 1961, Yale University Press, p. 11.

- IV. Representation of column matrices as geometric vectors.
- V. Transformations of the plane.
- VI. A special set of Research Exercises is appended with the hope that students will be introduced to real mathematical research.¹

The Ball State Teachers College Experimental Program was investigated for a content outline for a senior mathematics course. Their recommendation for a senior mathematics course was as follows:

Course title: Elementary Functions (a one semester course).

- I. Functions.
- II. Polynomial functions.
- III. Tangents to graphs of polynomial functions.
- IV. Exponential and logarithmic functions.
- V. Circular functions.²

This first semester course was similar to the content recommended by the SMSG Committee. The Ball State Teachers College Experimental Program further recommended the content that was in the textbook, Pre-Calculus Mathematics, published by Addison-Wesley.³ Ball State recommended plane and solid analytic geometry for the second semester course. The topics considered for the plane and solid analytic geometry course was as follows:

¹Ibid., p. 12.

²Ibid., p. 11.

³Ramon L. Avila, Ball State University, A personal letter received from Dr. R. L. Avila, April 8, 1970.

- I. Analytic Geometry In the Plane.
- II. Conics.
- III. Analytic Geometry In Space.
- IV. Other Coordinate Systems.
- V. Vectors.
- VI. Vectors In Three Dimensions.¹

The content given for the senior mathematics course was indicated by Dr. Ramon L. Avila when he stated:

These recommendations reflect the feeling of this department (the Ball State University Mathematics Department) that the four-year program in college preparatory mathematics at the high school level should be of a pre-calculus nature.²

V. MATHEMATICAL ITEMS DELETED

The following secondary mathematics committee reports were not given in complete detail: The University of Maryland Mathematics Project, Developmental Project in Secondary Mathematics of Southern Illinois University, Minnesota National Laboratory, New York State Mathematics Syllabus Committee, Boston College Mathematical Series, Mathematics Program at Phillips Exter Academy, The Advanced Placement Program of the College Entrance Examination Board, and The Report of the Cambridge Report Conference on School Mathematics. These committee recommendations were not given in complete detail for the following reasons: several programs advocated the teaching of the calculus a

¹Ibid.

²Ibid.

full year; several thought that an introduction to the first and second derivatives should be given; several had inadequate information available for the senior mathematics program; and several programs were working on material for the junior high school mathematics curriculum.

Calculus, at the senior level of high school instruction, was deleted for several reasons. The main reason was that the Mathematics Coordinator for the Davenport Public Schools, Mr. Harlan Goldsmith, indicated that the calculus should not be taught in the high school. Another reason given was that several mathematics curriculum committees stated the calculus was a college course only. The Commission on Mathematics of the College Entrance Examination Board had this statement concerning the teaching of calculus in the high school:

. . .Calculus is a college-level subject. A reasonable immediate goal for most high schools is a strong college preparatory mathematics curriculum that will have students ready to begin calculus when they enter college.¹

The CEEB further reported:

The commission does not recommend the inclusion of a course in calculus as a part of the normal high school curriculum. The average student cannot be adequately prepared for such a course in three years, and anything less than a full year course will ordinarily be time wasted, since it will not fit into any typical college program. Moreover, a course in calculus deals with ideas that are mathematically quite sophisticated, and mathematical maturity is absolutely essential.²

¹CEEB, "Modernizing the Mathematics Curriculum," op. cit., p. 14.

²Ibid., p. 13.

Another reason given for the exclusion of the calculus in the high school curriculum was that the teachers were often not adequately prepared to teach the calculus properly. The high school teachers often do not have the depth of understanding needed for adequate instruction. Furthermore, to teach the calculus properly, a teacher must know some theory of functions as well as advanced calculus.¹

Further instruction of geometry, as a course or as a unit, was not given great consideration. The instructional staff for the Davenport Public Schools indicated that they would not feel adequately prepared to teach any geometry beyond the analytical geometry level. Consequently, geometry was deleted as a possible unit of instruction.

The inclusion of the course of study of the trigonometry and analytical geometry was deemed essential because of the many duplications which appeared in the course outlines recommended by the Davenport Public Schools, and those provided by the other senior mathematics curriculum committees. The content for the semester trigonometry course was as follows:

Trigonometry

This course is a study of the properties and relations among trigonometric functions and circular functions.

Course of Study:

- I. Sets, relations, and functions.

¹George Grossman, "Advanced Placement Mathematics-for Whom?", The Mathematics Teacher, LV (November, 1962), 564.

- A. Real numbers
 - 1. Natural numbers
 - 2. Integers
 - 3. Rational numbers
 - 4. Irrational numbers
 - B. Open sentences
 - 1. Equations
 - 2. Inequalities
 - C. Cartesian product
 - D. Relations
 - 1. Domain
 - 2. Range
 - 3. Function
 - 4. Composition of functions
- II. Circular functions
- A. Periodic functions
 - B. Basic circular functions
 - 1. Cosine
 - 2. Sine
 - C. Properties of basic circular functions
 - D. Reduction formulas
 - E. Additional circular functions
 - 1. Tangent
 - 2. Cotangent
 - 3. Secant
 - 4. Cosecant
 - F. Identities and proofs
- III. Graphs of circular functions
- A. Graphs of cosine and sine
 - B. Graphs of other circular functions
 - 1. Asymptotes
 - C. Graphs of functions involving sine and cosine
 - 1. Amplitude
 - 2. Period
 - 3. Phase shift
 - 4. Location of graph

- D. Graphs of other functions
 - E. Graphing by addition of ordinates
 - F. Uniform circular motion and simple harmonic motion
- IV. Inverses of circular functions
- A. Inverses of cosine and sine
 - B. Inverses of other circular functions
 - C. Open sentences
- V. Trigonometric functions
- A. Angle measure
 - B. Trigonometric functions
 - C. Trigonometric identities
 - D. Tables for trigonometric functions
 - 1. Reference angles
 - E. Solution of triangles
 - 1. Law of sines
 - 2. Law of cosines
 - 3. Ambiguous case
- VI. Vectors
- A. Vectors and geometric representation
 - 1. Norm and magnitude
 - 2. Direction angle
 - 3. Vector addition
 - 4. Scalar multiplication
 - B. Basis vectors
 - 1. Linear combination
 - 2. Unit vectors
 - 3. Vector of scalar components
 - C. Inner product of two vectors
 - 1. Vector addition
 - 2. Scalar multiplication

- D. Applications
 - 1. Free vectors
 - 2. Forces
- E. Polar coordinates
 - 1. Graphs of polar equations
- VII. Complex numbers
 - A. As ordered pairs
 - B. Standard form
 - C. Graphical representation of complex numbers
 - D. Polar form of complex numbers
 - E. De Moivre's Theorem
- VIII. Infinite series and trigonometric functions (optional)
 - A. Limit¹

The content for the semester analytical geometry course was as follows:

Analytical Geometry

The purpose of this course is to relate the concepts of algebra and geometry and to lay the foundation of applied mathematics. It is a study of rectangular and polar coordinates in two and three dimensions.

¹Davenport Public Schools, op. cit., pp. 23-24.

Course of Study:

I. Inequalities

- A. Review of the basic axioms of the real number system
- B. Introduction of interval notation
- C. Using the properties of the real numbers
 - 1. Solving open sentences involving inequality
 - 2. Solving open sentences involving absolute value
- D. Solving inequalities involving polynomials using the number line

II. Relations, functions, and graphs

- A. Review of the definitions and notation involved in the study of relations and functions
 - 1. Domain
 - 2. Range
 - 3. Projections
- B. Use of symmetry and intercepts in graphing
- C. Finding the intersection of loci
 - 1. Algebraically
 - 2. Graphically

III. The straight line

- A. The distance formula
- B. The midpoint formula
- C. Slope of a line
 - 1. Parallel lines
 - 2. Perpendicular lines
- D. Equations for the lines
 - 1. Point-slope form
 - 2. Two point form
 - 3. Slope-intercept form
 - 4. Two-intercept form
 - 5. Parametric equations
- E. Point of division formula

- F. Distance from a point to a line
- G. Families of lines
- H. Angle between two lines
- I. Linear inequalities in two unknowns

IV. Vectors

- A. Vector equality
- B. Operations with vectors
 - 1. Vector addition
 - 2. Multiplication by a scalar
 - 3. Scalar product

V. The Conic sections

- A. Circle
 - 1. Equation of a circle
 - 2. Tangents to the circle
 - 3. Normals of the circle (optional)
 - 4. Families of circles
- B. Parabola
 - 1. Equations and definitions
 - 2. Tangents to the parabola
 - 3. Normals to the parabola (optional)
- C. Ellipse
 - 1. Equations and definitions
 - 2. Directions
 - 3. Tangents
 - 4. Normals (optional)
- D. Hyperbola
 - 1. Equations and definitions
 - 2. Directrices, tangents, and asymptotes
 - 3. Normals (optional)
- E. Translation of axes
- F. Rotation of axes

VI. Transcendental functions

- A. Review graphing of trigonometric functions

- B. Inverse relations and functions
- C. Exponential and logarithmic curves
- VII. Polar coordinates (optional if covered in trigonometry)
 - A. Graphing equations
 - B. Symmetry
 - C. Equations in polar and Cartesian coordinates
- VIII. Solid analytical geometry
- IX. Vectors in three dimensions (optional)¹

Since trigonometry was a first semester course and analytical geometry was a second semester course the previous year for these senior mathematics students, the content pertaining to trigonometry and analytical geometry was reviewed extensively in order to determine what topics should be placed in the senior mathematics course. The unit on circular functions was deleted from the CEEB proposed senior mathematics course since it was discussed in detail in the trigonometry course previously. Likewise, the SMSG, Ball State, and the UICSM unit on circular functions was omitted as a possible topic for the senior mathematics course.

The units on functions and relations from the CEEB and functions and polynomial functions from the SMSG were deleted since this content was covered rather extensively in intermediate and advanced algebra. Functions and relations were reviewed in the trigonometry and analytic

¹Ibid., pp. 25-26.

geometry course the following year for these students. This left approximately a one-half of a semester course for the senior mathematics course. The topics of logarithmic and exponential functions were considered for inclusion in the senior mathematics course, but they had been covered in advanced algebra so they were dropped.

Introduction to Probability and Statistics as proposed by the CEEB was considered next as a possible one-semester course. However, the coordinator of the mathematics curriculum for the Davenport Public Schools indicated that some of the items outlined by CEEB must be deleted.

The senior mathematics course was not to consist of a full semester of Probability and Statistics. Several reasons given for not having a semester course were: (1) college credit would not be given for this course, (2) no suitable high school textbook available for a full semester course, and (3) the content of the course would require a unit on the infinite series, usage of natural logarithms, and usage of a computer since many problems involved detailed calculations and a computer would be ideal for the solving of these problems.

A unit on the computer was considered, but it was rejected at this time because a course had not been taken by the instructional staff. A unit on the computer would be added as soon as a staff member received instruction on the computer and adequate facilities were made available (a room for the computers), within the school system.

The content for Probability and Statistics was reduced somewhat; consequently, the course was less than a semester in length. The

deletion of some of the content for Probability and Statistics as proposed by the CEEB was in accord with what other schools had been doing. This was indicated by Mosteller who had indicated that the Poisson distribution was avoided because of the infinite series since schools find it difficult to find adequate staff members.¹

Probability and Statistics was allotted approximately six to seven weeks of instruction. As a result the content of the CEEB proposed course of Probability and Statistics had to be revised to accommodate this arrangement.

The content of Introduction to Modern Algebra, as proposed by the CEEB, was then considered for the senior mathematics course. The system of rational numbers was covered rather extensively in the advanced algebra course as well as the properties for rational numbers. Topics involving inequality properties were covered in rather complete detail in the advanced algebra, trigonometry, and analytical geometry courses. The remaining content, as proposed by the CEEB, was then considered. However, it was determined that some of the topics for the Modern Algebra course would not be covered in as great a detail as was proposed by the CEEB. Therefore, the remaining content did not require a full semester.

¹Frederick Mosteller, "What Has Happened to Probability In the High School?", The Mathematics Teacher, LV (December, 1967), 824-831.

The Selected Topics as proposed by the CEEB were considered. The topic for probability was considered to be of importance and was to be included in the senior mathematics course. The extension of the topics for graphing of rational functions, systems of equations, coordinate geometry of three dimensions, and graphing of inequalities and absolute values were also considered. Since these topics were covered in rather complete detail in the advanced algebra, trigonometry, and analytical geometry courses they were deleted from the proposed senior mathematics course.

The UIGSM senior mathematics course advocated derivatives and antiderivatives as was indicated earlier in this chapter that the mathematics coordinator, Goldsmith, had stressed that calculus was not to be included in the senior mathematics course. Other reasons, as stated earlier, were that calculus was a college level course only, high school teachers were not adequately prepared, and that the high school students would be lacking in mathematical maturity.¹ Therefore, these two topics were deleted from this course content as well as circular functions, limits and continuity, and exponential and logarithmic functions. The topics remaining that were considered important for the senior mathematics course were as follows: (1) matrix algebra, (2) probability, (3) mathematical induction, and (4) mathematical structures.

¹CEEB, op. cit., pp. 13-14.

From the SMSG course content outline, the following items were deleted for possible usage in the senior mathematics course: (1) tangents to graphs of polynomial functions, (2) circular functions, (3) polynomial functions, and (4) exponential and logarithmic functions. Topics that were retained from the SMSG outline were the proposed course for matrix algebra and selected items from the various units involving functions.

From the Ball State Teachers College program the topic on space was deleted since it was primarily concerned with topics relating to analytical geometry. Elementary functions and their graphs, circular functions, and topics relating to trigonometry were considered to have been covered in trigonometry the previous year. Topical items that were considered of importance for possible placement in the senior mathematics course were as follows: vectors, systems of linear equations and determinants, mathematical induction, and combinatorial problems.

As a result of all these exclusions from each of the four mathematics programs, no complete senior mathematics course was left as established by these committees. However, the items that were left were certain topics on functions, probability, statistics, matrix algebra, mathematical induction, topics from Modern Algebra (abstract algebra) vectors, and complex numbers. These topics were then considered. It was determined, however, that some of these topics could be used for a full semester course and others for only several weeks of instruction.

The process of selecting the topics was further complicated by the fact that even though some of these topics were covered for a full semester, the student would still have to repeat the course in college. The reason for this is that college credit is not available at the Davenport Public Schools for courses taught in high school. It was determined, therefore, that a full semester would not be devoted to any one topic.

VI. THE CONTENT FOR THE SENIOR MATHEMATICS COURSE

What should be the procedure and content for the curriculum of the senior mathematics course? This question was considered in the following manner. The content that was selected was based upon the following items as stated by P. S. Jones:

1. What topics will best help students to understand the nature, role, and fascination of mathematics?
2. What topics will form the best basis for further studies by the students, whether they be motivated by necessity or by curiosity?
3. What topics are most necessary to the future citizen's ability to understand his culture and to function effectively in it?
4. Which of all these topics can be most effectively studied at the students' level of maturity and in the time available?¹

¹P. S. Jones, "The Mathematics Teacher's Dilemma," The University of Michigan School of Education Bulletin, XXX (January, 1959), 65-72.

The first unit proposed for the senior mathematics course was The History of Mathematics. This topic was to be used a unit for individual study by each student. Justification for this unit was presented from the following viewpoints. The unit would form a basis for the discovery approach to mathematics whereby the student would be allowed to discover some basic facts about the evolution of the various mathematical ideas as they occurred in history. In addition, the student would do individual research in an area of mathematics. A historical approach to mathematics was further indicated by Freitag and Freitag who stated:

A proper fusion of the history of mathematics with its teaching and learning can only produce desirable results. Some of the possible improvements are in deepening, fostering, and strengthening the outcomes of the traditional approach.¹

They also indicated that:

. . .The historical method is an asset to the attainment of mathematical objectives of both teaching and learning.²

They further stated that:

Any educator interested in improving the teaching of mathematics must consider a more intelligent use of historical material.³

The topics considered and included in the unit on the History of Mathematics were as follows: (1) chronology of pi, (2) the three

¹Herta T. Freitag and Arthur A. Freitag, "Using the History of Mathematics In Teaching on the Secondary Level," The Mathematics Teacher, L (March, 1957), 221.

²Ibid.

³Ibid., p. 220.

problems of antiquity, (3) the development of our number system, (4) contributions made by the Egyptians, Greeks, Babylonians, Hindus, and Arabians to the early development of mathematics, (5) men of mathematics (Euclid, Pythagoras, Archimedes, Euler, Fibonacci, Descartes, Heron, Gauss, Newton, Leibnitz, Cardano, De Moivre, Pascal, Bernoulli, Lagrange, Copernicus, Kepler, Fermat, Hilbert, and others), (6) the events and men that led to the development of the courses of algebra, geometry, analytical geometry, probability, trigonometry, and arithmetic.

It was also considered that some of the ideas suggested would be rather difficult for the students. However, it was determined that if many ideas and areas were discussed, it would allow the students to come into contact with as much history of mathematics as possible. It was considered desirable to allow the students to investigate an area or a man in as much detail as possible rather than to completely neglect the topic.

The inclusion of this unit on the History of Mathematics was best indicated by the statement made by Fehr which stated:

Indeed, compared with students who have no historical approach, those who do, show a deeper understanding of the principles and concepts involved, an ability to apply the principles to solving problems, a genuine appreciation of mathematics as an integral element of the culture in which it develops, and greater respect for knowledge as man's guide to improvement of his life.¹

¹ Howard F. Fehr, "Teaching High School Mathematics," Department of Classroom Teachers American Educational Research Association of the National Education Association, Article No. 9, (October, 1955), 8.

Topics that would be included in the course content for the History of Mathematics were those that were largely found to be unanswerable. Some of these topics were as follows:

1. Does there exist, at very least, one even number greater than two, which is not a sum of two primes?
2. Topology problem of Koenigsburg.
3. Goldbach Conjecture: That each natural even number greater than six, is a sum of two different primes.
4. Is there any odd natural number n for which the sum of all its natural divisors is $2n$?
5. Is it true that in the decimal expansion of $\sqrt{2}$, the digit one occurs an infinite number of times?¹

It was further indicated that no teacher would be expected to know all the material in this proposed unit. However, the teacher would be expected to know where the available material was located and to aid the student in his undertaking of his project for the research paper.

The second unit that was placed in the senior mathematics course was the study of the nature of mathematics. This unit was to include number fields, groups, and rings. Complex numbers were introduced and were to be discussed in detail since they had been introduced only briefly in either advanced algebra or trigonometry. The course content for this area is presented in detail later in this chapter. This unit

¹W. Sierpinski. "Enrichment Mathematics for High School," National Council of Teachers of Mathematics, 28th Yearbook, Washington, D.C.: 1963, 209-217.

had been advocated for inclusion in the senior course by the CEEB, the UICSM, and the Mathematics Coordinator for the Davenport Public Schools.

The third unit of instruction that was placed in the senior mathematics course was functions and relations. This unit was reconsidered and was finally placed in the senior mathematics course because some of the topics on functions and relations had been covered in the sophomore year of instruction and some completely different ideas on functions had been discussed in the junior year. Therefore, this unit of functions and relations was to consist of a brief but very thorough review of the definition of function and relation. The review was to include linear, quadratic, polynomial, circular, periodic, exponential, logarithmic, and inverse functions. These items were advocated by the CEEB, SMSG, UICSM, and Ball State Teachers College Mathematics curriculum committees for the senior mathematics course. The content is presented in detail later in this chapter.

Mathematical induction was the next unit placed in the senior mathematics course. The instruction of this unit was to include sequence and series. The definition of mathematical induction of sequences and series, and of arithmetic and geometric progressions was included. Proofs by mathematical induction were included. This unit was advocated by the CEEB, UICSM, and Ball State Teachers College for the senior mathematics course. A detailed account of this unit was included in this chapter.

Vectors was the next topic placed in the course outline. The definition of a vector was given, its usage and application was developed.

Also, addition and scalar multiplication were defined and discussed. Vectors, as a unit, were stressed by the SMSG in some form in their matrix algebra, the CEEB in their junior year of mathematics, and by the Ball State Teachers College program. The content was placed in the senior mathematics course since it was deleted by some instructors in the trigonometry because of lack of time. The detailed content of this unit appears in this chapter.

Probability and statistics were introduced in this course outline and followed the pattern established by the CEEB, several items from Ball State Teachers College, and UICSM. Some of the items deleted from the detailed outline established by CEEB were: (1) the law of continuity, and (2) the relation between two variables. This unit is presented in detail later in the chapter. The final topic that was placed in the senior mathematics course outline was Matrix Algebra. Among the items of importance that were stressed were the definitions of a matrix, the operation of matrices, using the matrices to solve problems, the definition of a determinant, the process of how to find a determinant, and the application of a determinant. The primary sources of information for this unit were SMSG and the Ball State Teachers College program for senior mathematics. The complete detail of the course content for Matrix Algebra was given in detail in this chapter under the course outline.

VII. SENIOR MATHEMATICS COURSE OUTLINE

This course is designed to lay the groundwork for the calculus. It will include the history of mathematics, the nature of mathematics, functions and relations, mathematical induction with sequences and series, vectors, probability and statistics, and matrix algebra.

The course of study:

I. History of Mathematics

A. Numeral Systems

1. Early counting and recording of numerals
2. Early computation in addition and multiplication
3. Selected problems

B. Contributions made by Early Civilizations

1. Egyptians
2. Babylonians
3. Greeks
4. Hindus
5. Arabs

C. Selected topics of interest

1. Chronology of pi
2. Three problems of antiquity
 - (a) Duplication of a cube
 - (b) Trisection of an angle
 - (c) Quadrature of a circle
3. Pythagorean Mathematics

D. Men of mathematics

1. Zeno 2. Aristotle 3. Euclid 4. Archimedes
5. Pythagoras 6. Ptolemy 7. Fibonacci
8. Copernicus 9. Ferrari 10. Tartaglia
11. Cardano 12. Galileo 13. Kepler 14. Napier
15. Briggs 16. Gunter 17. Fermat
18. Descartes 19. Pascal 20. Gregory
21. Newton 22. Leibniz 23. Bernoulli
24. De Moivre 25. Gauss 26. and other men as discovered by the students.

E. Contributions to the Various Branches of Mathematics

1. Algebra and arithmetic
2. Geometry (Plane and other)
3. Trigonometry
4. Analytical geometry
5. Probability
6. Calculus (if student interest demands it)¹

F. Word origin as they are encountered in the senior mathematics course.

G. Mathematical conjectures

1. Goldbach conjecture
2. Koenigsburg Bridge Problem
3. And other conjectures²

II. Nature of Mathematics

A. Introduction of groups

1. Preliminary definitions and assumptions
2. Definition of a group
3. Commutative (Abelian) Groups
4. Examples of non-commutativity
5. Subgroups
6. Group generators
7. Cyclic groups
8. Theorems about groups

B. Introduction to Rings and Fields

1. Definition of a ring
2. Theorems about rings
3. Integers (modulo n)
4. Assumptions concerning the real and complex numbers
5. Definition of a field
6. Theorems about fields.³
7. Ordered fields

C. Complex numbers

1. Definition of a complex number
2. Complex number as an ordered pair
3. Graphical representation of a complex number

¹Eves, op. cit., pp. xi-xvi, and 383-390.

²Sierpinski, op. cit., pp. 209-217.

³Flora Dinrines, Abstract Mathematical Systems (New York: Appleton-Century-Crofts, 1964), pp. vii-viii.

4. Four operations on complex numbers
5. Complex numbers and quadratic equations
6. Polar form of a complex number
7. De Moivre's theorem
8. Roots of a complex number

III. Functions and Relations

A. Definitions

1. Relation
2. Function

B. Types of functions

1. Linear 2. Quadratic 3. Polynomial
4. Circular 5. Periodic 6. Exponential
7. Logarithmic 8. Inverse 9. Other uses of functions

IV. Mathematical Induction

A. Definition of mathematical induction

B. Proofs by mathematical induction

1. Formulas
2. Discovered formulas

C. Definition of a sequence and a series

D. Progressions

1. Arithmetic
2. Geometric

E. Binomial theorem

F. Infinite sequences and series

G. Limit of a sequence

H. Infinite geometric series

I. Axiom of completeness

V. Vectors:

A. Definition of a vector

B. Properties of a vector. (Addition and scalar multiplication)

- C. Geometric representation of a vector
- D. Basis of a vector
- E. Inner product of two vectors
- F. Free vectors
- G. Vector application to forces. (Parallel and perpendicular forces included).
- H. Polar coordinates and graphs
- I. Polar coordinates and equations

VI. Probability and Statistics

- A. Definition of probability and statistics
- B. Statistics
 - 1. Random sampling
 - 2. Other sampling procedures
 - 3. Representation of the data (various types of graphs)
 - 4. Percentiles
 - 5. Definition: Mean, median, mode, quartile, etc.
 - 6. Measures of central tendency
 - 7. Deviations and its calculations
 - 8. Normal curve and application of the standard deviation
- C. Probability
 - 1. Introduction by "intuitive approach"
 - 2. Experiments with a (a) coin, (b) dice, (c) deck of cards
 - 3. Samples spaces. Examples and definitions
 - 4. Events and their probabilities
 - 5. Probabilities and sets
 - 6. Compulsory events
 - 7. Independent events and dependent events
 - 8. Conditional probabilities
 - 9. Random drawings and numbers and the use of the table of random numbers
 - 10. Permutations (linear and circular permutations)
 - 11. Combinations
 - 12. Binomial distribution

VII. Matrix Algebra

- A. Definition of a matrix
- B. Operation with matrices
 - 1. Equality 2. Addition 3. Multiplication
 - 4. Multiplicative inverses
- C. Using matrices to solve problems
 - 1. Introduction
 - 2. Triangulation method
 - 3. Formulation in terms of matrices
 - 4. Solution by means of matrices
 - 5. Diagonal method
 - 6. Matrix inversion
 - 7. Linear systems in general
- D. Linear transformations
- E. Algebra of matrices
- F. Determinants
 - 1. Definition
 - 2. Finding determinants
 - 3. Using determinants to solve problems
 - 4. Inverses
- G. Augmented matrix
- H. Using matrices to study the properties of
 - 1. Groups
 - 2. Rings
 - 3. Fields

This was a course content outline only. The detailed course outline was given in the Appendix B of the report. The detailed amount of time allowable for each unit was given by the following time table:

Suggested 180-Day Time Schedule

Chapter (or unit)	1	2	3	4	5	6	7	8
Number of days	20	15	40	15	15	40	35	?

The course was designed for a full year, and classes will meet daily for fifty-five minutes. Class time will be used for presentation of the material, and the assigned problems will be done outside of class time.

Unit eight was added only if, by some chance or other, the content for the first seven units were covered in complete detail and time were available for further course work. Unit eight was concerned with logic and methods of proof. The material for this unit was advocated by UICSM, CEEB, and SMSG. The main topics advocated were the usage of sets and the application of sets to the various areas of algebra. Appendix B contains an outline of this unit.

Unit VIII. Logic and Method of Proof

- A. Language and symbols
- B. Proofs
 - 1. Truth tables
 - 2. Syllogisms
 - 3. Indirect proof
 - 4. Law of detachment
- C. Sets applied to:
 - 1. Algebra
 - 2. Logic
 - 3. Boolean algebra

The main course, covering units one to seven, would be expanded or deleted in content as deemed essential by the instructor. The time allotment was only to serve as a guide. Several days should be allowed for the presentation and discussion of the oral and written reports. The time allotted for the individual reports would depend upon class size as well as the nature of the reports.

With the completion of the content for a senior mathematics course, the next item for the curriculum was the selection of a suitable senior mathematics textbook. However, the selection procedure for a textbook was lacking, as stated before in Chapter II. Consequently, a procedure had to be established for the selection of a senior mathematics textbook.

DEVELOPMENT OF THE TEXTBOOK SELECTION MATERIAL

I. INTRODUCTION

Textbooks occupy a unique position in the American Educational System. Many times the textbook largely determines the content and the organization of a specific course of study. Either fortunately or unfortunately, this is particularly true of the mathematics courses taught in the American schools (for both elementary and secondary levels of instruction). In most cases the content of the courses is dependent upon the textbook content. Consequently, the selection of a textbook is a very important task for a course of instruction. Increasing difficulty is encountered in the selection of an appropriate textbook because of the many textbooks available, the many varied mathematics programs being announced as the best series for adoption, and the many varied thoughts presented as to the theory and practice to be applied to a school situation for instruction. As a result of these items of difficulty, it was considered that the problem of textbook selection would be an increasingly complex situation for a teacher to make a valid selection.

In view of the importance and the difficulty of evaluating a textbook, it was considered extremely important to have a procedure available to select a textbook with as much objectivity as possible.

The material reviewed and the material available failed to establish the selection procedure desired for a senior mathematics textbook. Since a procedure was deemed essential, it was then decided to develop a selection process that could be used to select an appropriate senior mathematics textbook with as much objectivity as possible. It was deemed essential that the selection procedure be made with considerable care. As was stated by Atwan, in an article from the School Executive:

The kind of equipment selected and the method by which it is selected can influence directly the quality of education provided for the children.¹

Ideas for the actual content of the selection procedure were derived from several sources. The content for the evaluation of a textbook, according to the Des Moines Public Schools, should be concerned with the following items: (1) authorship and publisher, (2) approach and method, (3) content, (4) teachability factors, and (5) format.² The Davenport Public Schools indicated the following items as being of importance: (1) adequate diagnostic material, (2) drill on basic skills, (3) acceptable reading level in the explanations and in the word problems, (4) sufficient word problems of varying degree of difficulty, (5) attractive format-interesting illustrations, (6) is the modern approach being used, (7) the attractiveness of the book

¹A. M. Atwan, "Selecting Equipment: A Cooperative Enterprise," School Executive, LIX-LX (November, 1949), 59.

²Des Moines Public Schools, "Criteria for Textbook Evaluation," op. cit.

cover, (8) logical organizations, (9) concept development, (10) illustrations, and (11) stimulating content for further study.¹ This guideline for textbook selection, as furnished by the Davenport Public Schools, was an one-page score sheet with only the topics (1-11) given as indicated in the above listing for the Davenport Schools. Cedar Rapids Public Schools indicated that the content of the book was the most important item to be considered for the selection of a textbook.² They further indicated that they were developing a textbook selection procedure similar to the recommendations made by the National Education Association.³ The Allamakee County School District (Waukon, Iowa) indicated that the main points to be considered for textbook selection were as follows: (1) authorship, (2) philosophy, (3) content and organization, (4) method, (5) aids for teachers, (6) vocabulary, and (7) illustrations.⁴

Since the Cedar Rapids School District had indicated that they were considering a new evaluation procedure for the selection of a textbook and were going to develop their selection procedures according to

¹Davenport Public Schools, "Criteria for Textbook Evaluation."

²Cedar Rapids Public Schools, "Criteria for Textbook Evaluation."

³Ibid.

⁴Allamakee County School District (Waukon, Iowa), "Criteria for Textbook Evaluation."

the guidelines recommended by the NEA, it was determined that the selection procedures being developed in this study should also review the recommendations made by the NEA. The NEA recommendations are in a booklet titled: Guidelines For Textbook Selection. The recommendations made in this publication were as follows:

1. Innovation. To bring about fundamental changes in courses of study. Textbooks can help bring about improvements in content, organization, and teaching methods.
2. Up-to-date content. To insure accurate and up-to-date content by selecting textbooks that include new concepts, insights, and facts.
3. Provisions for Individual Differences. To select textbooks that stimulate pupils grouped heterogeneously and those organized by levels or tracks.
4. Grade-to-grade Development. To select those series of textbooks that provide for growth from grade to grade.
5. Course-of-study Correlation. To select textbooks that best correlate with local or state courses of study.
6. Instructional Assistance. To select textbooks with the kind of supplementary aids that will best help teachers reach their highest level of efficiency.
7. Inspiration and Growth for Teachers. To select textbooks that will encourage teachers to revise and improve their methods and that will inspire their work with the zeal and vigor that come from launching a new enterprise.¹

The textbook selection criteria were developed with the following facts to be considered in the actual development of the criteria:

¹Report of the Joint Committee of the National Education Association and the American Textbook Publishers Institute, "Guidelines for Textbook Selection," National Education Association (Washington, D.C.: 1969), pp. 23-24.

(1) the course of study, (2) the total school mathematics curriculum, and (3) the educational philosophy of the school system.¹ Ideas for the content of the textbook selection procedure had to be established which would involve the three above facts. The first item, (1) the course of study for the senior mathematics course, was developed in this chapter. The second and third items were already established by the school district and were simply restated in this chapter.

The actual content for the selection of a textbook was formed from the seven features established by the NEA in their booklet: Guidelines For Textbook Selection. These seven items were stated as follows: (1) possible innovations within the course of study, (2) the material was up to date, (3) provisions for individual differences, (4) grade to grade development of the content, (5) course of study correlation, (6) instructional assistance for the teacher, and (7) suggestions for the teacher to improve the method of instruction.² These seven recommendations were selected because they most closely related to the literature and score cards reviewed by the author. However, these seven items were only suggestions made by the NEA and did not give any specific items to observe when selecting a textbook.

The general over-all importance of any one of the seven items listed in the report made by the NEA could not be completely ignored.

¹Ibid.

²Report of the Joint Committee of the National Education Association and the American Textbook Publishers Institute, op. cit., p. 19.

In reference to one of these items, namely part (3), provisions for individual differences, it was deemed necessary that the textbook, as well as the curriculum content, must allow for individual differences within the classroom. As Smith noted in his report:

Even six pupils in a group, no matter how carefully selected, are still six individuals with different abilities, interests, rates, and so forth.¹

The importance of part (1), the possibility for innovation within a course of study; part (6), for instructional assistance for the teacher; and part (7), suggestions for the teacher to improve the method of instruction, was indicated by Leese, Frasure, and Johnson who stated:

Textbook publishers have made a decided contribution by increasing the flexibility of texts to encourage teacher initiative. Actually, many of the better books are now set up as initial and basic resource tools containing extensive and appropriate suggestions for the development of a variety of learning activities.²

Provisions for individual differences as well as the remaining three features, part (2), the material was up to date; part (4), grade to grade development; and part (3), a course of study correlation were stressed as being of utmost importance to the educational process in an article by Anderson who indicated the following:

¹Rolland R. Smith, "Provisions for Individual Differences," The Learning of Mathematics, Its Theory and Practice, The Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: 1953), p. 273.

²Leese, Frasure, and Johnson, Jr., op. cit., p. 46.

1. Selection of instructional material should follow a study of the program and its objectives.
2. The greatest extent possible, the choice of the material and the resources used should be the responsibility of those who use them.
3. The choice of the material and resources should be based upon the purposes, maturity, and the background of the group for whom it is intended.
4. The staff should cooperatively establish the criteria for the selection of the materials to be purchased for the school.¹

Anderson further stated that:

Too often books for use in the classroom are selected on bases other than the objectives to be achieved and the nature of pupils to be taught. The book may look teachable, it may have an attractive format; or the teacher may find that other teachers are using it. When materials are selected in these terms, objectives become empty statements.²

All of these outlines and recommendations were reviewed in detail in order to establish a procedure for the selection of a senior mathematics textbook. Upon reviewing these procedures, however, for selecting a textbook, it was decided that many different suggestions were offered. Within each suggestion, though, there was much subjectivity required in order to give an evaluation of a senior mathematics textbook. The content that was considered important for the development of a procedure for the selection of a textbook was formed from many and varied sources. Content finally selected for the evaluation procedure was as follows: (1) aims, (2) authorship and research,

¹Vernon E. Anderson, Principles and Procedures of Curriculum Improvement (New York: Ronald Press Company, 1956), pp. 326-331.

²Ibid.

(3) content and its organization, (4) methods, (5) teacher and student aids, and (6) special features included in the textbook.

Each area will be identified, explained, and its importance stressed by quoting a source of reference, some author, or a mathematics committee. The details for determining whether these items are attained by a given textbook will be given in the score card (or scoresheet). The usage of the score sheet is indicated in Chapter Four. Two textbooks are used for evaluation, and a recommendation is made for an adoption of the textbook evaluated highest according to the score sheet.

II. DISCUSSION OF THE TEXTBOOK SELECTION MATERIAL

Aims. In order to be an effective tool of education, the textbook must have goals that are comparative with the goals of the school district. Once the goals have been established by the school district, it is then possible to determine the curriculum and the material needed to aid in the instruction of a given course. It was suggested by Georges that the material presented should make a contribution to the central aims of the course and to the ideas involved in the instruction of mathematics to indicate that mathematics is a process of thinking.¹

The goals of the school district, for mathematics instruction, should be clearly stated prior to the selection of a textbook, and some thought should be given as to the type of textbook desired for the

¹Joel S. Georges, "Selecting a Textbook In Mathematics," School Science and Mathematics, LIII (May, 1954), 345-346.

given course of study. For the purposes of this report, it was assumed that the goals for the school district had been established; consequently, the goals need not be established by the textbook committee. The goals would be stated only by the committee so as to coincide with those of the school district.

As one evaluates the textbook, it is hoped that the aims of the textbook are clearly stated and coincide with or compare favorably with those established by the school district for the given course. If the aims of the textbook are not clearly stated, it is hoped that the teacher will be able to establish aims from the content of the textbook to compare favorably with those established by the school district.

Authorship and research. An item of considerable importance to be employed in an analysis of a textbook is the author of the book. Hopefully, the author is competent in the subject area in which he is writing and that he had had some experience as a teacher in the subject area. If an author lacks competency in the subject area of his writing, then difficulty may be encountered by the teacher regarding the written material by the author. Hall-Quest indicated that a free lance author with no concept of the subject area or regard for concerted opinion may not necessarily be wrong in the presentation of the subject matter, but it constrains the teacher to investigate carefully the author's reason for his views.¹ Hall-Quest further implied that if the author

¹Hall-Quest, op. cit., p. 184.

has broad training and broad preparation, then the teacher of the material will be more inclined to accept his statement as safe and constructive for educational purposes.¹ If the material has been tested in the classroom, then the teacher can have reasonable assurance of its value for instructional use. As was noted earlier, by Cronbach, the material of a textbook must withstand the usage in the classroom. The textbook evaluation is not complete until it is used in the classroom.²

It was further indicated that the author of a textbook has a responsibility to society. The author can influence society by his writings. His product, in the final analysis, is an instrument of power.³

In most cases, the publisher lists the credentials of the author. The listing, hopefully, includes the type of academic and professional or educational training which the author has had. This training of the author and his professional experience deserve some consideration.⁴

Concluding the item for authorship and research, the writings of the author should conform to the course of study, the total mathematics curriculum, and the educational philosophy of the school district.

¹Ibid.

²Cronbach, op. cit., p. 188.

³Roger Bierstadt, "The Writers of Textbooks," Text Materials In Modern Education. (A series of chapters pertaining to text materials) (Urbana, Illinois: University of Illinois Press, 1955), p. 127.

⁴Ibid., p. 185.

If the authorship is questionable, then the textbook would be considered undesirable for possible use by the school district.

Content and organization. The most important item to be considered in the selection of a textbook is the book's content. The book, for all practical purposes, is of no value unless it is opened and used. Once the book is opened, the content of the book becomes the prime consideration for the evaluation of the textbook. The content is the major determining factor as to the relative importance of the book. This was indicated by the NEA in a research memo stating:

The most important thing to judge in evaluating a textbook is the material contained in it and the way it is presented.¹

The importance of the content of the material was given in Chapters I and II, when curriculum committees indicated that the content of the mathematics curriculum must be developed first. Then a textbook can be selected. A further indication as to the importance of the content of a mathematics textbook was given by the numerically weighted score card used by the Des Moines Public Schools for the evaluation of elementary mathematics textbooks. Out of a possible 1000 points they had 450 points for the content of the mathematics textbook.²

¹National Education Association, Research Division, Research Memo, 1962, No. 27, p. 5.

²Des Moines Public Schools, op. cit., pp. 1-5.

When determining the content of the textbook, many questions must be asked in regard to the content. As a result of the review of literature on textbook selection, the following summary of questions was found to be of importance for the selection of a senior mathematics textbook:

1. Does the material present itself as being readily readable for the grade level intended?
2. Does the content agree with the mathematical objectives of the school district?
3. Does the author's selection of topics agree with the proposed senior mathematics course?
4. Does the content agree with national mathematics curriculum committee's recommendations and suggestions?
5. Does the content coincide with the educational level of the student?
6. Does the content have accuracy in the presentation of the definitions and problem exercises?
7. Does the content present up-to-date terms for the reading material and the exercises?
8. Does the content contain mathematical rigor in the proofs and usage of the language?
9. Does the content present itself in terms of new learning theories and new approaches to problem solving?
10. Does the content allow for individual differences in the problem exercises?
11. Does the content present itself as isolated topics, or is the content presented in a continuous pattern of mathematical thinking?
12. Does the content relate to the real world in terms of possible uses and practical problems?

13. Does the content relate to the real world in terms of possible uses and practical problems?¹

The importance of each item listed will be stated as a result of a committee's report or some author's viewpoint. The content of the book must be up-to-date (7) as was indicated by the NEA and the American Textbook Publishers Institute.² Further indication of the importance of up-to-date content was given in a report titled, Administration of the School Library, whereby material was considered to be out of date if the following conditions existed:

1. The material no longer conforms to the present and prevailing ideas of information on that topic.
2. The general rule for technical books is that the books are out of date in ten years or less. (Some say five years because of the rapid technological processes of the past decade.)
3. The material is to be replaced if a book exists that is entirely new and offers newer and better material.³

The subject matter in the textbook should show some evidence of relation to a need, (item 12) and to be consistent with the level of the maturity of the student, (item 5). The need for the subject should be shown in the context or in the introduction of the concepts presented.⁴

¹A summary of questions from the score sheets of the school districts of Allamakee, Cedar Rapids, Davenport, and Des Moines.

²NEA; and The American Textbook Publishers Institute, op. cit., p. 19.

³Administration of The School Library, A college course in Library Science, Course No. L.S. 320, Iowa State Teachers College, 1948, pp. 1-2.

⁴Houtz, op. cit., p. 253.

Material presented should relate to the needs of the student and should be presented in some form in the textbook to the student.¹ The maturity of the student is important. That the content must be readable and understandable by the student was further indicated by the score sheets from Des Moines² and Cedar Rapids.³

The vocabulary, (item 1), used in the context of the textbook should be easy to understand and yet sufficiently dignified to be a challenge to the student. In addition, the vocabulary should be sufficiently rigorous to prevent vagueness in the student's thinking.⁴ In addition, the vocabulary should be such that it is readable, accurate, and elegant. If at all possible, the material should be interesting and entertaining at the same time as it is informative.⁵

If the content, (item 2) does not agree with the outlined mathematical objectives of the school district, the textbook could be rejected. As indicated earlier in the paper, the curriculum should be developed first. Then the textbook can be selected not the reverse!

The selection of topics, (item 3), by the author should coincide

¹Cedar Rapids School District, op. cit., pp. 1-4.

²Des Moines School District, op. cit., pp. 1-5.

³Ibid., pp. 1-4.

⁴Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, op. cit., pp. 389-417.

⁵Department of Public Instruction, op. cit., p. 3.

with those of the school district as much as possible. If too many topics are deleted from the author's selection in comparison to the school districts mathematics outline for the school, then the teacher would have to provide supplemental material. In addition, the selection of the topics, (item 4), should compare favorably with those recommended by the national mathematics committees.¹

Accuracy, (item 6), of the data presented in the textbook must be as reliable as possible.² Furthermore, the information, the problem, and the definitions presented should be accurate and complete.³ The Davenport School District's directive to the Senior Mathematics Curriculum Committee stated that if too many errors are evident, the textbook was to be rejected.⁴ The general importance, pertaining to the accuracy of the content of a textbook, was quite ably given by Clements who stated:

At the present time, factual errors in textbooks are regarded as almost inexcusable, in the case of some, if not in the case of

¹Reeve, op. cit., pp. 601-609.

²John Addison Clement, Manual for Analyzing and Selecting Textbooks (Champaign, Illinois: The Garrad Press, 1942), p. 22.

³Proceedings of a work conference, Philadelphia Conference, June 2-3, 1955, Co-sponsored by ALA Committee on Intellectual Freedom and the Public Libraries Division, "Book Selection," The PLD Reporter, American Library Association, Chicago, Illinois, 1955, p. 51.

⁴Davenport Public Schools, op. cit.

all of the subject groups. Furthermore, with pertinence, it may be rightly insisted that original data presented by the authors themselves, or data borrowed from other sources, should be as accurate and reliable as it is possible for them to be, in the light of supporting evidence.¹

Mathematical rigor, (item 8), in a textbook refers to the nature of the development of the arguments and the kind of justification that is used in a proof. The rigor of a textbook may have much to do with the understanding of the subject matter. Usage of proofs cannot be overemphasized at this grade level of mathematics. The emphasis on proofs is good for the student since it should contribute to his mathematical maturity.² Further indication of the importance of rigor and its related operations was indicated by the CEEB in the following statement:

The understanding of the deductive method as a method of thought. This includes the abstraction of mathematical models from the outside world. . . . It also includes the ideas of axioms, logical reasoning, methods of proofs, and the relationship between proved theorems and physical reality.³

New ideas for learning and problem solving, (item 9) was placed here, as well as in the area of methods, since the new textbooks are written in a variety of ways. Several textbooks considered for evaluation

¹Clement, op. cit., p. 22.

²A report to the National Council of Teachers of Mathematics, "An Analysis of New Mathematics Programs," National Council of Teachers of Mathematics (Washington, D.C.: 1963), p. 48.

³CEEB, op. cit., p. 4.

placed a continued emphasis on routine manipulation of problems. The traditional textbooks, as indicated by Meder, had the following features:

1. Routine manipulation of problems in an artificial situation.
2. Topics were often dated and of no importance to the current student.
3. Topics were too elementary for current use.
4. Topics presented as isolated items and not related to the other areas of mathematics.¹

The importance of the new approaches to mathematics have indicated that the modern programs in mathematics are as effective as traditional programs in developing traditional mathematical skills.² One of the advantages of the new approach to mathematics was indicated by Rosskopf who revealed that mathematics must stress concepts since the skills needed may disappear entirely or change, but the mathematical concepts are likely to be used in the future by the student.³

Rosskopf further stated:

¹Albert E. Meder, Jr., "Proposals of the Commission on Mathematics of the College Entrance Examination Board," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), 19-26.

²Holland Payne, "What About Modern Programs In Mathematics?", The Mathematics Teacher, LVIII (May, 1965), 424.

³Myron F. Rosskopf, "Proposals of the Commission on Mathematics of the College Entrance Examination Board," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), 1.

. . .Furthermore, a student who has well developed mathematical concepts is more adaptable to the sort of life he might meet in the future.¹

A further indication of the importance for considering the modern approach to mathematics as presented in the mathematics textbook was noted by Spencer who stated:

Children should be led to make their own investigations and to draw their own inferences. They should be told as little as possible and induced to discover as much as possible.²

Furthermore, various national committees have recommended that the modern approach be used in mathematics. These recommendations have been noted previously in the paper. The importance of the new approaches used in mathematics presentation of the content has been that there is an emphasis of "relationships" in the mathematics textbook.³

Individual differences, (item 10), within the classroom must be taken into consideration when one selects a textbook. No classroom is completely homogeneous; the exception would be if there is only one student in the class. As had been indicated earlier in an article by Smith, the class size may be ever so small, yet individual differences exist.⁴ These individual differences, within a class, create a wide

¹Ibid.

²Herbert Spencer, Education (New York: D. Appleton Century Company, 1920), p. 120.

³Georges, op. cit., p. 348.

⁴Smith, op. cit., p. 273.

range of abilities. The Secondary-School Curriculum Committee of NCTM indicated the following in regard to individual differences within a class:

The question arises as to what mathematics the pupils should study. The answer appears to be: the same mathematical structure and concepts, but varying in amount, in complexity, in depth, and in manner of organization and presentation in order to be consistent with the pupil's ability and his past achievement. All pupils may eventually study the same algebra (mathematics), but not at the same time and at the same rate, to the same depth of understanding, with the same application, or necessarily in the same sequence.¹

Further provisions for individual differences are being recognized by textbook publishers. Textbook publishers have organized their textbooks as a resource tool containing extensive and helpful suggestions for a variety of learning activities within the classroom.² The NEA and the American Textbook Publishers Institute also deemed it essential to consider the importance of individual differences within a class.³

Problems and exercises in the senior mathematics textbook are an important part of the content of the textbook and are concerned with the learning of the material in the course in a continuous pattern of mathematical thinking. Consequently, careful attention should be given to these items. The problems should be organized so that they allow for

¹Secondary-School Curriculum Committee of the National Council of Teachers of Mathematics, op. cit., p. 408.

²Leese, Frasure, and Johnson, Jr., op. cit., p. 46.

³National Education Association, op. cit., pp. 23-24.

individual differences, whether they be of social or economic significance. It is necessary to stimulate thought amongst all students and to aid the students in obtaining some initial mastery of the subject material.¹ It was also considered of some importance if the problem sections contained an oral exercise section as an aid, for both the student and the teacher, to obtain greater initial understanding of the concepts presented in a given lesson.

Mathematical content, (item 12), of the textbook should be related to the real word. This statement was indicated by Kline who further indicated that mathematics is not an isolated subject.² To help develop the notion that mathematics is not an isolated subject in the school, one should try to present concrete examples and to minimize abstractions in mathematics.³ Kline further indicated that:

Mathematics is the prime instrument for understanding and mastering the physical world. This is the chief value of mathematics in a liberal education.⁴

Problem exercises should be available within the textbook that are within the experiences of the student and the problems connected

¹Harry E. Houtz, "Teachers Can Help Evaluate and Select Textbooks," Elementary School Journal, LVI (February, 1956), 250-254.

²Morris Kline, "A Proposal for the High School Mathematics Curriculum," Mathematics Teacher, LIX (April, 1966), 323.

³Ibid.

⁴Ibid., p. 330.

with life situations. In addition, the problem exercises should be of practical usage for the student.¹

Another item of importance (item 13), perhaps one of the most important, was the benefits for the student that are presented in the textbook. Within this area of benefits to the student one would include: the readability of the material, the problem exercises, the accuracy and precision of the definitions and content, the allowances for individual differences, and the allowances for intellectual stimulation. A textbook should be selected that communicates with the student as the most important item in the educational process.

In an article by Atwan, although not directly related to textbooks, indicated the following fact about education:

The kind of equipment selected and the method by which it is selected can influence directly the quality of education provided for the children.²

It was also indicated that the textbook should be written to the student and should talk to the student. This fact was indicated by Clements who stated, "The textbook as a whole should be adapted to the needs of the learner for whom it is intended."³

¹Frank A. Jensen, Current Procedure In Selecting Textbooks (Philadelphia and Chicago: J. B. Lippincott Company, 1931), pp. 130-136.

²Atwan, op. cit., p. 59.

³Clement, op. cit., p. 14.

The significance of having the content established for the student is of tremendous importance for the high school. In planning the curriculum, one must bear in mind three items as indicated by the CEEB:

1. It is not known exactly what career any student will follow;
2. It is not known exactly how the mathematical needs of various occupations will develop in the years ahead;
3. Although much mathematical instruction aims at future usefulness, mathematics for its own sake is a valuable part of the general education of any future citizen.¹

The importance of having the material prepared and presented for the student was stated by SMSG in the following objectives:

First we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and the structure of mathematics. Second, the mathematics program must attract and train more of those students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.²

Methods. The manner in which the content is presented to the student is of importance. Questions asked as to how the subject matter is to be presented are indicated in the following manner:

1. How shall we teach our mathematics?
2. As a set of manipulative skills?
3. As a set of ideas?

¹Commission on Mathematics College Entrance Examination Board, Objectives of the Commission on Mathematics of the College Entrance Examination Board, College Entrance Examination Board, 1957, p. 3.

²School Mathematics Study Group, op. cit., pp. 3-4.

4. As a set of structures, either intuitively or rigorously developed?
5. As a physical study headed toward an abstract science?
6. Should we stress "doing the mathematics" and "understanding the concepts and development" equally or one more than the other?
7. As algebra, geometry (plane and solid), trigonometry, analysis, etc., or as one continuous development of mathematics?¹

The method of instruction presented in a textbook cannot be separated from the subject matter present in the textbook. As was indicated by George in the following summarized statement:

Psychological principles of a book should involve the presentation of mathematics as a useful method of thinking in terms of situations close to the student and not as a collection of tricks. The individual differences provided for in the textbook are accomplished by the careful grading of exercises, optional material, and supplementary exercises and activities.²

Discovery of mathematical ideas as a means of instruction has been recommended by various mathematics curriculum reports.³ Textbook materials prepared by MSG have used the discovery technique in the presentation of their material. In addition, the UICSM textbook material uses the discovery approach as a method of instruction in their

¹Fehr, op. cit., p. 425.

²George, op. cit., pp. 345-351.

³Secondary-School Curriculum Committee of the National Council of Teachers of Mathematics, op. cit., pp. 389-417.

mathematics textbook.¹ This was indicated by McCoy who stated that the UICSM had the following major items:

1. A consistent exposition of high school mathematics is possible.
2. Manipulative skills, though necessary, should be primarily used to emphasize basic concepts.
3. The language read in textbooks or heard from the teacher should be as unambiguous as possible.
4. The mathematics materials should be organized in such a fashion that students would have abundant experiences in discovering generalizations.²

This fact was indicated earlier in the paper but has been restated because of its over-all importances to the method of instruction in a textbook.

Another method used in the learning process is the axiomatic approach to mathematics. The axiomatic approach is quite similar to the discovery approach. The discovery approach asks questions at each step of the learning process. As a result of these questions asked, the student is able to see what he is doing and why he does it in this particular manner.³ An axiomatic approach has the student assess the various ideas he has discovered in relation to past learnings of the content presented.

¹Kinsella, op. cit., p. 28.

²McCoy, op. cit., pp. 12-18.

³George, op. cit., p. 345.

Vocabulary used in the textbook for the method of instruction and presentation is of vital importance. The vocabulary must be such that the student can understand the material. In regard to the vocabulary of the textbook, Smith and Heddens stated:

The readability levels of materials to be used for the textual purposes would seem to be a very crucial consideration. Surely a student cannot be expected to make efficient use of these materials if he is incapable of reading them.¹

They further indicated that if the reading material is too difficult, then the teachers will find it necessary to provide additional instructional reading material in mathematics. In addition, the difficulty of the reading material would require more than the usual amount of explanation of the material.²

Teacher and student aids. Teacher and student aids available in the textbook are of importance to both since they use the textbook. Aids that are of importance to the student and teacher would include reference material, points to be noted at the beginning of the chapter, a review of the important facts at the conclusion of each chapter and a chapter test.³

Special aids for the student would include special exercises to stimulate intellectual interest and reasoning at the close of each

¹Smith and Heddens, op. cit., p.393.

²Ibid., pp. 393-394.

³Suggestions from the score cards of Allamakee, Cedar Rapids, Davenport, and Des Moines School Districts, op. cit., pp. 1-5.

chapter. Practical applications of the content presented would be an aid for further learning for the student.¹

Teacher aids would include a review of the experimental studies used by which the author's conclusions were reached. A summary of the best pedagogical knowledge available on the subject material should be presented in the textbook. Books of reference would be of benefit for the teacher and, also, for the student. The teacher's edition should include the answers for all exercises, a clear explanation of new terminology and new processes, enrichment materials, tests, special teacher's commentary, and suggestions for teaching of the material.²

Special features. Special features of the textbook include the table of contents, an index, variation in the printed material to emphasize important items to be learned by the student, illustrations, special tables such as (square and square roots, logarithm tables, trigonometric functions, logarithm table of trigonometric functions, cube and cube roots, statistical tables, and a listing of all the formulas used in the textbook), and any other features the teacher would deem essential as an instructional aid for the student.³

Included within the special features of the textbook are the special tests (cumulative and chapter tests) and cumulative reviews. The special features of a textbook would also include any special teaching aids such as overlays for the overhead projecture or overlays

¹Ibid.

²Ibid.

³Ibid.

used within the textbook to illustrate a specific learning situation. In general, special features of a textbook are important to the overall appearance, usefulness, and effectiveness of the textbook.

IV. OUTLINE OF THE SELECTION PROCEDURE

The suggestions and recommendations made from the writings of various writers and curriculum committees are presented in a summarized outline which is usable for selection of a textbook by either a teacher or a committee. Content of the outline is to be used for the selection of a senior mathematics textbook. The textbook selection outline is as follows:

- I. Are the aims of the textbook stated by the author and are they such that they coincide with or compare favorably with those of the school district?
 - A. Are the aims explicitly stated or implied in the following manner within the textbook:
 1. the preface of the textbook?
 2. introduction of the textbook?
 3. contents throughout the textbook?
 - B. Are the aims in direct conflict with those established by the school district?
 - C. Or, are the aims not given at all?
- II. Does the author have classroom experience in the same area as that of the textbook which he has written?
 - A. Are the teaching experiences from:
 1. Elementary schools?
 2. Secondary schools (Junior or senior high school)?
 3. College or university levels of instruction?
 - B. What educational training has the author had?
 1. Is the educational training in the subject area of the textbook?
 2. Other training.

- C. Has the content of the textbook been tested?
 - 1. By the author in the classroom?
 - 2. With one's colleagues?
 - 3. Or by the author and his colleagues?
 - D. Does the content, as presented by the author, conform to the content recommended by various curriculum committees?
 - E. What other articles has the author written?
- III. Does the content of the textbook include the following features?
- A. Does the material present itself as being readily readable for the grade level intended?
 - 1. Does the vocabulary present itself as being easily understood in the:
 - (a) definitions?
 - (b) explanations?
 - (c) problem exercises?
 - B. Does the content of the textbook agree with the mathematical objectives of the school district?
 - C. Does the author's selection of topics agree with the proposed senior mathematics course content?
 - D. Does the author's content agree with the national mathematics curriculum committee's recommendations and suggestions?
 - E. Does the content coincide with the educational level of the student?
 - F. Does the content have accuracy in the presentation of the definitions and problem exercises?
 - G. Does the content present up-to-date terms for the reading material and the exercises?
 - H. Does the content stress mathematical rigor?
 - 1. Rigor stressed in proofs.
 - 2. Rigor stressed in the usage of the terms.
 - 3. Foundations properly made for proofs.
 - 4. Proofs clearly indicated and developed.

- I. Does the content present itself in terms of new learning theories and new approaches to problem solving?
 1. Does the content emphasize understanding as well as computational skills?
 2. Does the content use new approaches in the problem exercises?
 3. Does the content encourage the student to make a generalization?
 4. Does the content develop basic principles for the course?
 5. Does the content develop the definitions in terms of the new approaches to mathematics?
 6. Does the content indicate the historical development of mathematics as an aid for the understanding of mathematics?
 7. Are different eras of mathematical development indicated?
 8. Are important men, causes, and countries mentioned as an aid to the understanding of the concepts in mathematics?

- J. Does the content allow for individual differences in the problem exercises?
 1. Does the content indicate problems that are unsolvable as challenges for the student?

- K. Does the content present itself as isolated topics or is the content presented in a continuous pattern of mathematical thinking?
 1. Are the topics presented in a logical sequence or are they presented as an isolated area of mathematics?
 2. Are certain items indicated as being workable in a certain area but not possible to work in other areas?
 - (a) $x^4 - 25$ factorable in integers
 $(x^2 - 5)(x^2 + 5)$
 - (b) $(x^2 - 5)(x^2 + 5)$ factorable as $(x - \sqrt{5})(x + \sqrt{5})$
 $(x^2 + 5)$ over the real numbers.
 - (c) $(x - \sqrt{5})(x + \sqrt{5})(x^2 + 5)$ factorable as $(x - \sqrt{5})(x + \sqrt{5})(x + i\sqrt{5})(x - i\sqrt{5})$ over the complex numbers.

- L. Does the content relate to the real world in terms of possible uses and practical problems?
 1. Are the problems artificial, or are they related to other areas such as:
 - (a) science?

- (b) social sciences?
 - (c) technical problems of the various technical trades?
 - (d) business?
- M. Does the organization of the content benefit the student?
- 1. Are the topics organized to the student's expanding needs, interests, and increasing mathematical ability?
 - 2. Are the terms and ideas from previous grades redeveloped in new situations?
 - 3. Are the terms and ideas from previous grades used to develop new learnings in a spiral effect of content at each grade level?
- IV. Are the methods of instruction such that:
- A. The material is presented in terms of the student's experiences and understanding?
 - B. The textbook is communicating with the student?
 - 1. Textbook is for the student.
 - 2. Student can read the textbook as if the textbook was a self-teaching unit.
 - C. The discovery and axiomatic approaches are used in the textbook?
 - D. Individual differences are considered when:
 - 1. new material is presented?
 - 2. problems and exercises are given?
 - E. Exploration opportunities exist for the student to develop certain relationships regarding the content?
 - F. The new material presented proceeds from the concrete to semi-concrete to the abstract?
 - G. Review of the important material exists for each chapter?
 - H. Tests exist for each chapter and periodic cumulative tests are given?
 - I. Accuracy exists in the explanation of the material and in the problem exercises?
 - J. Ample exercises exist to develop an understanding of the material presented?

- V. Are there student and teacher aids available in the textbook such that:
- A. The textbook can be used as a reference book by the:
 - 1. student?
 - 2. teacher?
 - B. Separate textbooks exist for the student and the teacher?
 - C. Oral exercises exist as an aid for the learning of the new material in the student and teacher's edition of the textbook?
 - D. The teacher's edition contains:
 - 1. Specific suggestions for teaching the content.
 - 2. Suggestions for handling individual differences.
 - 3. Adequate suggestions for enrichment materials.
 - 4. Tests for each chapter and cumulative tests.
 - 5. A suggested time table indicating the time suggested for each chapter.
 - 6. A detailed solution key for the problem exercises in the textbook.
 - E. The student's edition contains:
 - 1. Review exercises for each chapter.
 - 2. Sample chapter tests.
 - 3. Self-explanatory explanations of new concepts.
 - 4. References made from previous topics and those that follow so that a continuous pattern is evolved for the student.
 - 5. Answers to selected exercises in the student's edition.
- VI. Are there any special features in existence in the textbook that is under investigation?
- A. Does the textbook have:
 - 1. A table of contents?
 - 2. An index?
 - 3. A variation in the printed material to emphasize important items to be learned by the student?
 - 4. Special mathematical tables such as:
 - (a) square and square root tables?
 - (b) cube and cube root tables?
 - (c) logarithms tables?
 - (d) trigonometric functions for the sin, cos, tan, cot, sec, and csc?
 - (e) logarithm table for the six trigonometric functions?
 - (f) statistical tables (standard deviations)?

5. A listing of all the important formulas used in the textbook in a single listing in the textbook?
- B. Is there a listing of the important items at the end of each chapter?
- C. Other features deemed essential by the investigating teacher or committee to be listed by them in their evaluation of a given textbook.

In the outline just completed, suggestions were developed from the various score cards and score sheets reviewed and from the review of literature. Questions were used as a method of presenting the outline. Answers for each question will enable the teacher to determine the book that he considers most beneficial for the school district.

As an aid for the teacher in using the outline, a rating scale was developed and was given in Table I. The table contains the same main topics as the outline. In some cases, subtopics were included as an aid and others combined, reorganized, or deleted. An explanation is given if the reasons for variations within the subtopics were not apparent. Several items are mentioned twice. That is, the item is under the area of content and organization as well as under the area of methods. The reason for this is the importance of the item to the textbook for the senior mathematics course.

Exploratory material was listed under the comments by the evaluator of the textbook. The student's edition was placed first then the teacher's edition with the outline. This interchange was made because the teacher's edition may or may not exist for evaluation.

TABLE I

RATING SCALE FOR SENIOR MATHEMATICS TEXTBOOKS

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
I. Aims: (6 points)			
A. <u>Aims are stated.</u>			
The aims are stated and are in complete agreement with those of the school district.	The aims are stated, but are not in complete agreement.	The aims are stated but are for the most part in disagreement	The aims are not stated.
()	()	()	()
B. <u>The aims are recognizable in the content.</u>			
The aims are clearly recognizable and in complete agreement with those of the school district.	The aims are recognizable but not in complete agreement.	The aims are not clearly stated and are in disagreement.	The aims are not recognizable.
()	()	()	()
II. Authorship and Research: (15 points)			
A. <u>Classroom experience.</u>			
Teaching experience in mathematics at the high school level.	Teaching experience in junior high school or at the collegiate level.	Teaching experience at a level other than the secondary level.	No teaching experience.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>II. Authorship and Research: (Continued)</u>			
<u>B. Educational training.</u>			
Major training in the area of the content of the book.	Some training in the area of the content of the textbook.	Educational training only.	No training.
()	()	()	()
<u>C. Content tested in the classroom.</u>			
Content of the book has been successfully tested by the author and his colleagues.	Content has been used as experimental units only by the author and his colleagues.	Content has been based upon experience and observation, but not successfully tried in the classroom.	Content has been written without regard to classroom use or classroom observations.
()	()	()	()
<u>D. Content conforms to various national curriculum committees.</u>			
Content in complete agreement.	Most topics are in agreement.	Some topics included from the national committees.	No topics included from the national committees.
()	()	()	()
<u>E. Authorship.</u>			
Has written several acceptable articles and books in mathematics	Has written some articles and a textbook. Their content acceptable.	Has written articles only.	Has no experience in writing.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>III. Content and Organization: (75 points)</u>			
<u>A. Readable for the grade level intended.</u>			
Material is presented as being readily readable and understood in the given definitions, explanations, and problem exercises.	Content is readable in most areas.	Content is adequate, but certain areas cause difficulty in understanding the material.	Not readable for the grade level intended.
()	()	()	()
<u>B. Vocabulary.</u>			
Terms which are introduced are of significance. Terms are introduced and are used.	Terms introduced but not always used. Some terms are defined but are insignificant.	Some terms introduced but not defined, used, or are of any significance.	Nothing defined or explained.
()	()	()	()
<u>C. Content agrees with the districts proposed mathematics curriculum.</u>			
Complete agreement with the school districts proposed curriculum.	Several items not included but the majority of the items agree with the school district's recommended course.	Many items are not included in the textbook.	None of the items are in agreement with those of the school district's course outline.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>III. Content and Organization: (Continued)</u>			
<u>D. Proposed content in agreement with national committee recommendations.</u>			
Complete agreement.	Most items in the textbook agree with national recommendations.	Several items in agreement, others partially covered, and the rest not recommended for the given level of instruction.	No items in agreement.
()	()	()	()
<u>E. Accuracy of the content.</u>			
Accuracy is present in the definitions, explanations, and exercises.	Accuracy is present in most of the items presented.	Some inaccuracies are present in the explanations, definitions, and problem exercises.	Many inaccuracies are present in the content of the book.
()	()	()	()
<u>F. Up-to-date content.</u>			
Content is up-to-date and uses up-to-date procedures for presentation of the material.	Most items are up-to-date. Several dated topics or articles included.	Contains items that are recommended for omission. Several items acceptable.	Content is dated and has been recommended for omission.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>III. Content and Organization: (Continued)</u>			
G. <u>Mathematical rigor emphasized in the following:</u>			
1. <u>Proofs.</u>			
The development of proofs follow a logical argument and is there justification for the processes? Are the reasons indicated and the proofs demonstrated appropriate for the level of maturity of the student? ()	Proofs given but some items are deleted in the logical process of the proof. ()	Proofs are given but many proofs are left to be either proven by the student or simply accepted as they are presented. ()	The theorems are given, but the author indicates that the proofs are trivial and obvious. No proofs revealed. ()
2. <u>Precision of the language.</u>			
Precise language used in the definitions, explanations, and problem exercises. Precise language to be used by the students in their responses and in their proofs. ()	Precision expressed in most areas of the text. Content contains some precision. ()	Precision of the language not stressed but is evident in the book. ()	No precision of the language expressed. ()
3. <u>Foundations have been made for direct proofs before they are introduced.</u>			
Foundations are laid and clearly indicated so that the development of a proof is easily made. ()	Foundations are laid and indicated by the teacher. Development of a proof is made with assistance from the teacher ()	Foundations made for some proofs and ignored by others. Proofs need some explanation by the teacher for greater understanding for the pupil. ()	Foundations are almost non-existent. Proofs must be developed by the teacher since the content does not reveal what is to be done or proven. ()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
4. <u>Foundations have been made for indirect proofs before they are introduced.</u>			
Indirect proofs have been introduced carefully. Foundations for their introduction have been made in detail.	Indirect proofs introduced, explained, and illustrated in most cases.	Indirect proofs introduced but require some time for their development for understanding for the student.	Indirect proofs mentioned only. If to be introduced the teacher must develop the entire process of indirect proofs.
()	()	()	()
5. <u>Are alternate proofs indicated for the proofs?</u>			
Alternate proofs are mentioned and some of the more important alternate proofs are given in detail for the student.	Some alternate proofs are indicated or mentioned. Several alternate proofs are given.	Alternate proofs mentioned. The details of the alternate proofs are indicated for only several proofs.	Alternate proofs mentioned or simply not given.
()	()	()	()
6. <u>Does the context for the proof clearly indicate where a proof or its demonstration starts and ends?</u>			
Clearly indicated for every proof.	Usually indicated.	Several carefully readings of the proof are needed to determine the completion of the proof.	Not indicated.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
H. <u>New learning theories and new approaches to problem solving are indicated in the content.</u>			
1. <u>Understanding stressed.</u>			
Content emphasizes understanding as well as computational skill.	Understanding emphasized but lacking in some areas.	Computational skill stressed. Some understanding stressed in some areas	Understanding is forsaken. Computational ability stressed only.
()	()	()	()
2. <u>Discovery approach indicated.</u>			
Content presented with the discovery approach being used as means of understanding.	Most items introduced use the discovery approach.	Many items are simply stated. Little discovery approach used.	Concepts and definitions are simply stated.
()	()	()	()
3. <u>When certain facts are established for two space, are the students encouraged to consider whether these properties are applicable to three space?</u>			
Each major area, where such generalizations are possible, encourages the student to transfer the applications to three space.	Generalizations are made for most areas and the students are encouraged to make the generalizations for three space.	Occasionally, generalizations are mentioned and students encouraged to apply it to three space.	Generalizations are made only. Or the generalizations developed only once and then never repeated.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
4. <u>Development of the basic principles for the course.</u>			
Mathematical rigor is stressed throughout the book. Proofs, definitions, and problem exercises indicate mathematical rigor.	Rigor stressed in definitions and some proofs. Problem exercises usually stress the rigor.	Rigor stressed in some of the first few definitions and proofs. Later proofs ignore the desired rigor.	No mathematical rigor stressed.
()	()	()	()
5. <u>Development of definitions in terms of the new approaches used in mathematics.</u>			
The definitions are developed by the use of examples and generalizations. Definitions are amply defined and stated.	Most definitions are clearly and accurately stated. Several definitions are simply stated and no examples given for the use.	Definitions are clear, but seldom used. Some inaccuracies exist in the definition itself or it is not used correctly. Definitions are usually stated and not developed by examples or problems.	Definitions are simply stated. No development used for the definition given.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
6. <u>Does the author emphasize various areas of historical background for the major topics in mathematics?</u>			
Historical development stressed for the major areas. History used as an aid for understanding the mathematics. ()	Historical development mentioned only. ()	Several historical items mentioned or indicated ()	No mention is made of the historical development of the topics. ()
7. <u>Does the author indicate the various eras of mathematical development?</u>			
Ancient, medieval, middle, and modern ages indicated when possible. ()	Reference made to one or two eras only when possible. ()	Reference made only to an era when possible. ()	No mention is made. ()
8. <u>Does the author indicate the important men, developments, and countries in the development of mathematical concepts and ideas?</u>			
All items mentioned where possible. ()	Several items mentioned. Some references deleted. ()	Mentioned several times only. ()	No mention is made. ()
I. <u>Individual differences provided for in the textbook.</u>			
1. <u>Exercises are indicated for their difficulty.</u>			
Each exercise has the difficulty of the problems indicated. A sufficient number of exercises exist to develop an understanding of the material. ()	Individuals differences indicated but some problems are of questionable difficulty. Some exercises need supplemental material to account for individual differences. ()	Individual differences indicated but many exercises require supplemental material. ()	No provisions for individual differences. ()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
2. <u>Content indicates problems that are unsolvable or a valid proof has not been derived. (These problems are offered as challenges to the student.)</u>			
Problems from various areas of mathematics are offered as challenges to the student. Many problems and proposed solutions are offered the students.	One or two problems are mentioned. Some areas have several problems indicated.	One or two problems indicated. No possible solutions indicated.	None are indicated.
()	()	()	()
J. <u>Content organization.</u>			
1. <u>Content presented in a logical sequence.</u>			
Content in a logical pattern. Content within a topic area, logically organized.	Content must be rearranged somewhat.	Considerable rearrangement necessary to follow a logical pattern.	No logical pattern present. Topics are completely isolated units of instruction.
()	()	()	()
2. <u>Are certain problems indicated as being workable in one area but not workable in another area?</u>			
Example: $x^4 - 25$ factorable over:			
Integers. $(x^2 - 5)(x^2 + 5)$	Real numbers. $(x - \sqrt{5})(x + \sqrt{5})(x^2 + 5)$	Complex numbers. $(x - \sqrt{5})(x + \sqrt{5})(x - i\sqrt{5})(x + i\sqrt{5})$	
Clearly indicated and stressed in most cases. Questions are asked of the students in regard to possible restrictions on the operations of certain problems.	Indicated and illustrated. Questions occasionally asked of the student	Indicated and questions asked of the students on occasion.	No mention is indicated for any restrictions.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
------------------	-------------	-------------	---------------------

III. Content and Organization: (Continued)

K. Associations made with the real world.

Each individual unit makes some association to the real world. References made to science, business, or other areas.	Some items not related, but most have some association to the real world.	Few items are related to the real world. Topics are simply presented with no reference made as to their usefulness.	No associations presented as an aid to the usefulness of mathematics.
()	()	()	()

L. Content beneficial to the student.1. Material is presented for the student.

Material is presented to the student as material for the student and as material to be used by the student.	Most of the material is presented for the benefit of the student.	Some items are for the benefit of the student and other items are simply stated.	No benefits given or indicated.
()	()	()	()

2. Spiral approach used for redevelopment of previous terms in a new situation.

Frequent references made to previous content learned and how it applies to the new situation.	Some references made and how they apply to new situations.	References made only.	No references indicated.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>IV. Method of Instruction: (21 points)</u>			
<u>A. Presentation of the textbook material.</u>			
The material communicates directly with the student. Textbook is a self-teaching unit.	Communication is apparent, however, some explanation is needed by the teacher.	Explanation of the material is needed for the understanding of the content. Teacher must explain a great deal of the material.	Much supplementary material needed for a detailed understanding.
()	()	()	()
<u>B. Discovery and axiomatic approaches used for the presentation of the material.</u>			
Discovery and axiomatic approaches used in the explanation, definitions, and the problem exercises.	The two processes are used in most of the material presented for consideration. Some explanations need extra explanations.	The two processes are used in some of the cases. Most cases, a great amount of explanation is needed for the presentation of the material.	The traditional approach is used in most phases of the material.
()	()	()	()
<u>1. Discovery approach.</u>			
()	()	()	()
<u>2. Axiomatic approach.</u>			
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
IV. Method of Instruction: (Continued)			
C. <u>Abstract Material.</u>			
The abstract material is presented in a logical pattern for the student. Material proceeds from the concrete to the semi-concrete to the abstract material.	Material is usually presented in a logical manner.	Abstract ideas are simply stated. Little development is given for the abstract ideas.	Abstract material is stated and no development is given for the content.
()	()	()	()
D. <u>Reviews and tests.</u>			
Periodic reviews and tests are given. Cumulative reviews and tests are indicated throughout the textbook.	Reviews and tests are usually given for each chapter and each teaching unit.	Chapter tests or chapter reviews given only, not both.	Neither chapter tests or chapter reviews given.
1. <u>Reviews.</u>			
()	()	()	()
2. <u>Tests.</u>			
()	()	()	()
E. <u>Problem exercises.</u>			
Ample exercises given with adequate instructions given for each problem set.	Ample exercises given, but some supplemental material is required for greater understanding of the content.	Exercises given, but supplementary material is needed for individual differences.	Few exercises given and individual differences within the exercises are void.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>V. Student and Teacher Aids: (39 points)</u>			
<u>A. References.</u>			
Adequate references are given for the student and teacher at the end of each chapter. ()	References are usually given for the teacher and the student. ()	References given but not in detail. References given only for some chapters. ()	No references given. ()
<u>B. Separate textbooks.</u>			
Teacher and student editions are separate. ()	Same textbook. Same additional information given for the teacher's edition. ()	Same textbook. References available for the teacher. ()	Same textbooks. No additional references. ()
<u>C. Student's edition.</u>			
<u>1. Review exercises and sample tests for each chapter are given.</u>			
<u>(a) Chapter review exercises.</u>			
Contains review exercises, oral exercises, chapter reviews and tests, and vocabulary listings for each chapter. ()	Contains most of these items, but not in great detail. ()	Contains a chapter review or chapter tests only ()	No chapter tests or chapter reviews given. ()
<u>(b) Sample tests.</u>			
()	()	()	()
<u>(c) Oral exercises.</u>			
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
V. Student and Teacher Aids: (Continued)			
2. <u>References made from previous topics discussed to those that follow so as to develop a continuous pattern of mathematical thought.</u>			
Frequent references are made and the student is asked questions in regard to these references. Questions are asked of the student to recall previous material discussed.	References are made and the student is asked questions on occasion.	Some references are made, but not very frequently.	No mention is made of the previous topics discussed.
()	()	()	()
3. <u>Selected problems (odd or even numbered problems) have the answers given in the student's edition of the textbook.</u>			
Every problem set has answers for odd (or even) numbered exercises.	Answers for only certain problems.	Answers for only word or else computational problems.	None.
()	()	()	()
D. <u>Teacher's edition.</u>			
1. <u>Suggestions for teaching the material are indicated for the teacher in the teacher's edition.</u>			
Identical to the student's textbook but contains specific suggestions for teaching each topic, suggestions for individual differences, suggestions for supplementary material and answers are in the textbook.	Contains some of these items but not in as great a detail.	Few suggestions given.	No additional information available other than the textbook used by the student.
(a) <u>Teaching suggestions.</u>	()	()	()
()	()	()	()
(b) <u>Enrichment materials.</u>	()	()	()
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>V. Student and Teacher Aids: (Continued)</u>			
2. <u>The solution key contains a detailed solution for all the word problems and written exercises in the textbook.</u>			
Every problem is worked in detail. ()	Most of the problems are worked in detail. ()	Only the word problems are worked in detail. ()	Answers provided only for the problem exercises. ()
<u>E. Special features of the teacher's edition.</u>			
The teacher's edition is keyed to the student's textbook. Special chapter tests are available. A time table for teaching the content exists.	Special items are keyed only. Some answers are given. A few tests exist. A time table exists.	Several items are available.	No special features exist.
1. <u>Annotated teacher's edition.</u> ()	()	()	()
2. <u>Tests exist.</u> ()	()	()	()
3. <u>Suggest time table exists.</u> ()	()	()	()
<u>VI. Special Features: (15 points)</u>			
<u>A. Table of Content.</u>			
Table of content given in detail with all items of importance given. ()	Table of contents given, some important items given. ()	Table of contents given. ()	No table of contents. ()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
VI. Special Features: (Continued)			
B. <u>Variation in print.</u>			
Important items are readily noticed by a different colored print or style of writing.	Most of the important items are indicated.	Some items are indicated with a different print or colored ink.	No variation exists.
()	()	()	()
C. <u>Special tables.</u>			
Table of squares and square roots, cube and cube roots, common logarithms, trigonometric functions, logarithms of the trigonometric functions, and statistical tables.	Most of the tables given. Several omitted.	Square and square root tables given. Logarithm tables given.	No tables given.
()	()	()	()
D. <u>Formulas listed:</u>			
A complete and detailed listing of all the formulas is given as used in the textbook.	Most of the formulas are given.	Basic formulas are given. The rest of the formulas are omitted.	No formulas are given in a separate listing.
()	()	()	()

TABLE I (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>VI. Special Features: (Continued)</u>			
E. <u>Other features.</u>			
Style, readability, textbook size, study aids, and other items of educa- tional importance.	Most of the items stated are given.	Few aids are given. Few special features.	No special aids and few features of importance are given.
()	()	()	()
<u>VII. Comments to be used for other areas of importance or consideration:</u>			
(8 points)			

A check mark placed in the parenthesis under the appropriate column for each item considered was provided as an aid to evaluate a textbook more efficiently. The maximum number of points available for each major area is indicated in parenthesis with the major heading. A condensed one-page outline with only the major headings and their maximum point values are indicated in Appendix B.

Within the outline several items were indicated directly or indirectly more than once. These items were indicated in this manner because of their importance or impossibility of completely separating or distinguishing the item as a single entity.

The last item, additional comments, was included since the evaluator(s) may have used the textbook in the classroom, and this would be an aid for the evaluation of the textbook. Other comments may be included in this section as deemed essential by the evaluator.

Four items were used for the rating of each item given in the outline. A rating of three was given if the textbook was in complete agreement. A rating of two was given if most of the items indicated for each area were desirable. A rating of one was given for each area if only a few items for each area were acceptable, and a rating of zero was given if the area under consideration was found unacceptable. The rating scale in the outline included possible statements to be considered for specific items in the outline. If the statements were in complete agreement with the rating scale statements, then a rating

of three was given. However, if the statements were not acceptable, then a rating of zero was given.

Within the outline, the first answers given for each item would be the most beneficial to the actual rating of the textbook. Any variation from this would decrease the numerical rating of the textbook for that area. The numerical values selected for the outline were those that would be most beneficial to the outline for the selection of a textbook. Chapter IV contains two textbooks rated according to the given outline. The rating of the two textbooks was given as an aid to illustrate the usage of the developed outline for the selection of a textbook.

CHAPTER IV

DEMONSTRATION OF THE ESTABLISHED OUTLINE

I. INTRODUCTION

The study of literature was made in Chapter II concerning the content of the senior mathematics course and the selection procedure for a senior mathematics textbook. Chapter III had this information organized and developed into a senior mathematics course. The second part of Chapter III had the selection procedure for a textbook organized and developed into an outline to be used for the selection of a senior mathematics textbook. Since the outline for the selection of a textbook had not been used, the following demonstration of its usage was given as an aid to the evaluator in the evaluation of a senior mathematics textbook.

Two textbooks were selected for evaluation using the established outline from Chapter III. The first textbook selected was: Fundamentals For Advanced Mathematics by Glickman and Ruderman.¹ Hereafter, this textbook will be referred to as textbook A. The second textbook selected was, Modern Introductory Analysis by Dolciani, Beckenbach, Donnelly, Jurgensen, and Wooten and was referred to as textbook B.²

¹Glickman and Ruderman, op. cit.

²Mary P. Dolciani, Edwin F. Beckenbach, Alfred J. Donnelly, Ray C. Jurgensen, and William Wooten, Modern Introductory Analysis (Boston: Houghton, Mifflin Company, 1967).

The selection of textbook A and textbook B was that both books had been recommended rather highly by NSF Summer Institutes participants at Drake University during the 1968 and 1969 Institutes. A selection of any other type textbook for the proposed senior mathematics course was rejected because the textbook stressed a considerable amount of material to calculus, analytic geometry, trigonometry, or other topics not satisfactorily associated with the proposed senior course. This chapter contains a detailed evaluation of these two textbooks using the proposed outline for textbook selection. A discussion of the important items for each of the six major topic headings was given, and several comments are made in regard to each of the textbooks in the last part of the evaluation report.

The first major topic to be considered in evaluating a textbook, as established in the outline from Chapter III, was the topic of aims. As was indicated in Chapter III, the aims established for the mathematics curriculum were: (1) to achieve an appreciation for the development of mathematics, (2) to develop an interrelation among the various branches of mathematics, (3) to apply mathematics to the sciences, (4) to use mathematics as a tool, (5) to develop mathematical skills and concepts, (6) to provide intellectual challenges for each student, (7) to develop mathematical competence, and (8) to develop a favorable attitude toward mathematics.¹

¹ Davenport Public Schools, op. cit., p. 1.

With these aims established for the senior mathematics course, the two textbooks were examined and evaluated according to the established outline. The rating of each book will be given in the outline, and the reasons for the rating will be given in the following paragraphs. In each case, Book A will be rated first; then the reason for its rating will be stated. Book B will be rated secondly and the reason for its rating will be given in the succeeding paragraph. If the two books have the same rating and the reasons for the rating are similar, then the discussion of the rating will be given in the same paragraph.

A. AIMS

A. Aims stated.

Book A. (1) The aims of the books are given in a brief introduction to the course. The introduction of the book indicated that the book would be used to teach a variety of sequences of a pre-calculus level course. The sequences of the topics would be adapted to the school's curriculum, to the level of ability of the student, and to the preference of the instructor.

Book B. (3) The aims of the book are explicitly stated in the introduction of the textbook, and these aims are similar to those established for the senior mathematics course. The aims indicated in the book's introduction were to develop an understanding of the role of logic in the deductive systems of mathematics, to recognize the computational aspects of mathematics, the application of

mathematics, to develop an appreciation for mathematics, to prepare for future mathematics courses, and to achieve unity within the various topics of mathematics.¹

B. Aims Recognizable from the Content.

Book A and B. (2) Numerous references are made for the application of sets, probability, vectors, and exponential and logarithmic functions to the various sciences. Considerable stress is placed upon symbolism as a means of communication within these two. Definitions are precisely stated and explained in detail with acceptable vocabulary for this grade level of instruction. Emphasis is placed on the logical structure of mathematics in the proofs, the definitions, and the explanations given for the various topics in the two books. Frequent reference is made from one topical area to another topical area. The various properties of vectors are given, and then references to their applications are made in the area of complex numbers, trigonometry, and the analytic geometry of three-space. Problems and exercises are numerous, but no indication is made for the difficulty of the exercises in Book A. Book B has numerous exercises, and the difficulty of these exercises is clearly indicated.

¹ Dolciani, Beckenbach, Donnelly, Jurgensen, and Wooten, op. cit., (Teacher's Edition), p. 3.

B. AUTHORSHIP AND RESEARCH

A. Classroom Experience.

Book A. (1) It is indicated that the authors are members of the mathematics department of a high school, but no indication is made that they are actually teaching in the high school.

Book B. (2) Several of the authors have had junior and senior high school classroom experience and have received recognition for their outstanding teaching abilities. Other members of the writing staff for this book have had experience at the collegiate level of instruction.

B. Educational Training.

Book A. (0) The educational training is not indicated. No reference is made as to whether they have had any advanced educational training either.

Book B. (3) The authors are indicated as being either a professor, a dean, or a master instructor. The positions that each member of the writing staff have is indicated. The school where the authors are employed is also indicated.

C. Content classroom tested.

Book A. (2) The content of the textbook has been tried on an experimental basis in the classroom by the colleagues of the two authors. Some of the items have been the result of insights offered by the colleagues of the two authors.

Book B. (1) No indication is given if the content of the book has been tried in the classroom on a direct or an experimental basis. It is indicated, however, that the content has been developed as a result of a careful study of the many recommendations made by various mathematics curriculum study groups. The content developed by the various study groups has been tried; consequently, a (1) rating was given.

D. Author's Content Conforms to Various National Curriculum Committees.

Book A. (1) The book makes no reference to any use of the content proposed by the various mathematics study groups. However, if one investigates the book, one realizes that the content is adaptable to the various study group recommendations.

Book B. (3) The authors explicitly state that they have developed the course as a result of a careful study of the recommendations made by the various study groups. They further indicate that they have utilized recent developments in the learning and the teaching of mathematics.

E. Authorship (Previous Articles).

Book A. (0) No references are made or implied as to previous authorship.

Book B. (3) The authors of this textbook have written other textbooks for high school instruction. These authors have written previous books in this series for algebra I, geometry,

and modern algebra and trigonometry. It is further indicated that the members of the writing staff helped in the writing of SMSG materials.

C. CONTENT AND ORGANIZATION

A. Readability.

Book A. (1) The readability of the material for the presentation of the various topics is quite detailed. The explanations are quite detailed in length and require considerable comprehension for the senior high school student. Considerable amount of symbolism is used in the reading material.

Book B. (3) The material is presented in a readable manner, and the definitions and explanations are given with a minimum amount of reading required by the student. The usage of symbolism is kept to a minimum in the context of the book.

B. Vocabulary.

Book A. (1) In the first chapter the authors try to define everything possible. Pages 8 and 9 contain a listing of nine definitions. They defined admissible and permissible values, but these two definitions are not that significant to be stressed as specific definitions to be learned. In other words, the authors take a considerable amount of explanation to stress a point. In the explanation of the content they include the usage of many mathematical symbols within the explanation. This method of presentation may be acceptable; but for the student in the course, it may require an excess amount of

time of reading in order to obtain an understanding of the explanation presented. The author mentions the word "slope" on page 144 in the explanation of the material but has not mentioned or defined the word. He does, however, refer to a page of reference which is in the succeeding portion of the book. References are made to trigonometric terms on page 145 for inverse relations, but trigonometry has not been discussed or reviewed as of yet. In the definition for slope, pages 163-164, the authors use the letter "a" rather than the standard notation which has the letter "m". This notation may be acceptable but could cause confusion to the student if this specific change of notation is not indicated by the instructor. Later in the book, page 317, the author indicates the change in notation from "a" to "m" for the slope of a line within the reading material of the text.

Book B. (3) No difficulty was encountered in reading the material. Definitions are presented in concise terms; undefined terms are acceptable; the explanations given for proofs and sample exercises are concise.

C. Content in agreement with the district's curriculum.

Book A. (1) The book does include topics for functions and relations, mathematical induction, vectors, probability (only), exponential and logarithmic functions, and some reference to the historical development of some of the topics in mathematics. However, the book does not include any references for the nature

of mathematics (as developed for this school district's curriculum), for statistics, and for matrix algebra. The authors have topics on limits, the derivatives, its application, and the definite integral. These last three topics were not given in the district's proposed curriculum and were to be excluded as possible units of instruction. A considerable amount of supplementary material would be needed for the units on historical mathematics, nature of mathematics, statistics, and matrix algebra.

Book B. (2) This book included all of the district's proposed curriculum topics; however, the topics of statistics and matrix algebra were only briefly mentioned and not given a detailed discussion as desired by the district. The unit for the history of mathematics was not given as a unit, but the authors did list fourteen men of mathematics and their contributions to mathematics. The listings of the men of mathematics was a one-page summary for each man. Supplementary material would be required for the units involved with the history of mathematics, probability and statistics, and matrix algebra.

D. Content in agreement with the national committee's recommendations.

Book A. (2) Most of the topics were in agreement with the combined national committee's recommendations for a possible senior mathematics course. The exception was the last three chapters of the book where the authors included topics from the calculus. Calculus, as was indicated in Chapter II and III, was not acknowledged as a high school course.

Book B. (3) The proposed topics of instruction from this book were in agreement with the combined national committee's recommendations for a senior mathematics course.

E. Accuracy.

Book A and Book B. (3) Book A had one error present in its explanation of probability and the final answer. The error was on page 283, where the authors had $\frac{4}{52} \cdot \frac{3}{51} = \frac{3}{676} \underline{\underline{=}} .0044$. The correct solution is $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221} \underline{\underline{=}} .0045$. However, both books, upon careful investigation, had accuracy present in their definitions, explanations, illustrations, charts, and in the problem exercises. If numerous inaccuracies are present, they would have to be discovered as one actually used the two textbooks for classroom instructions.

F. Content up-to-date.

Book A. (3) Book A gives the Greek's geometrical point of view for the conic sections as well as the analytic approach to the conics. It develops the trigonometric functions in terms of the unit circle and the usage of the radian measure rather than the older form of defining the trigonometric functions in terms of the triangle. However, the book does make reference to the trigonometric functions in terms of the right triangle.

Book B. (2) The analytic approach to the conic sections is given, but no reference is made to the development of the conics as viewed by the Greeks. The unit circle and radian measure are used

for the development of the trigonometric functions. References for usage of the trigonometric functions in the triangle are given. However, the book spends considerable time on solving the six parts of a triangle using the trigonometric ratios. This time is debatable but perhaps beneficial. Reference is made to polar coordinates in the same chapter. This does help unify the various uses of the trigonometric functions. Investigation revealed no out-of-date material in the content, illustrations or problem exercises.

G. Mathematical rigor.

1. Proofs

Book A and Book B. (3) Both books give adequate rigor in their proofs and adequate justification for each process in the development of the proofs. In Chapters 3 and 5 of Book A, the proofs and examples are given in rather complete detail for the benefit of the reader. For Book B, Chapters 3 and 4, give a rather detailed demonstration for the proofs and examples for the benefit of the reader. Both books have some proofs given in detail, but not all the reasons are given for the complete derivation of the proofs. This is apparent in the chapters involving trigonometry and the conics.

2. Precision of the language.

Book A and Book B. (3) Both books use rather precise language and symbolism in the presentation of the proofs, definitions, and examples. However, of the two books, Book A uses greater precision of the language in its presentation than does Book B. Book B, however,

does not make its presentation of definitions, proofs, and examples as difficult to read as does Book A.

3. Foundations made for the introduction of the direct proofs.

Book A and Book B. (3) Both books give a rather detailed explanation of how to verify the solutions of sample exercises in their presentation of the material prior to the first formal proof. Many definitions are given prior to the first formal proof, and examples of how the definitions can be used in working various exercises are indicated in the first two chapters of each book. Book A has its first formal proof on page 20 while Book B gives its first indication of a formal proof by establishing an outline for a formal proof on page 24. The first theorem to be given and proven in detail in Book B is on page 40.

4. Foundations well developed before indirect proofs are indicated.

Book A. (3) The book develops the role of indirect proofs by giving many examples and indicating how to prove the examples wrong by contradiction and counter-examples. The first formal development of an indirect proof is done in a very detailed pattern. The proof outline is done adequately, and little trouble is encountered in understanding the processes of an indirect proof.

Book B. (1) In comparison to Book A, the development of an indirect proof is done rather abruptly and with no detailed

foundations developed for an indirect proof. The development for an indirect proof is adequate. However, in comparison to Book A, the development is not as well developed or as detailed as would be desirable.

5. Alternate proofs given for some proofs.

Book A. (2) Several proofs are indicated as having an alternate proof available. On page 64, two proofs are given for the same theorem; on pages 97 and 98, two different proofs are given for the same theorem. Other alternate proofs are indicated for either theorems or problem exercises in the book.

Book B. (0) No alternate solutions to either proofs or problem exercises were found in the investigation. However, if one were to use the book for instruction, one might find alternate solutions for a proof and a problem.

6. The development and completion of proofs and sample exercises are indicated.

Book A. (2) The development and completion of the proofs and sample exercises are usually indicated quite clearly. However, one must read the demonstration for the proofs and sample exercises to determine the completion of the demonstration.

Book B. (3) In comparison to Book A, which had an adequate demonstration for the development and completion of its proofs and exercises, Book B rates an even higher rating, because it uses the therefore (∴) notation to indicate the completion of its proofs

and sample exercises. Also, when numerical values are the answer for sample exercises, the book clearly indicates this by writing "answer" adjacent to the final solution.

H. New learning theories and computational skill stressed in the content.

1. Understanding and computational skill stressed in the content.

Book A and Book B. (3) Detailed explanations and illustrations are given as an aid for understanding of the material. Ample exercises are given to stress the understanding and computational skills needed for the course.

2. New approaches used in the problem exercises.

Book A. (2) The explanations are detailed, but they do not ask enough questions as to why or how the results were attained. The problem exercises, in certain problem sets throughout the book, ask questions of the students in regard to a specific problem in the problem set. Examples of some of the questions asked of the student in regard to a specific problem are, pages 313 and 314, "Do the points determine a triangle? Would it make sense to say that one of these three points lies between the other two? If so, which one?"

Book B. (2) The explanations are detailed and ask questions of the student such as:

(a) Why?

(b) Can you explain why something appears to be as it is indicated?

(c) Do you see why something appears as it is indicated?

(d) Can you provide an alternate solution or short cut?

These types of questions are indicated on pages 349-351 and appear frequently throughout the book. However, the problem exercises assigned to the students do not have this developmental technique stressed as frequently as was done in Book A.

3. Students encouraged to make generalizations.

Book A and Book B. (2) As was indicated in the previous two paragraphs of this section, both books encourage the student to make generalizations. Book A encourages generalizations in its problem exercises while Book B encourages the generalizations to be made from its sample exercises. In each book the discovery approach is used for the development of the generalizations to be made by the student.

4. Development of the basic principles for the course.

Book A and Book B. (3) Each book stresses the desired mathematical rigor for the course, the necessary understandings and computational skills needed for an adequate mastery of the content, and the encouragement for the students to draw their own conclusions.

5. Development of definitions in terms of the new approaches used in mathematics.

Book A and Book B. (3) Each book gives the definition in clear and accurate language, and the development for the numerous definitions is detailed enough for the student to understand the definitions and terms used.

6. Content uses the historical approach for the development of mathematics.

Book A. (1) The book makes reference to the historical approach to mathematics in the conic sections on page 412. The authors credit the Greeks for their study of the conics and indicate that Descartes, Khayyam, Desargues, Pascal, Kepler, Wallis, Newton, and others studied the conics intensively. Probability is indicated as being of some historical importance on page 262. Boolean Algebra is indicated as being developed by George Boole, page 95, but no further reference is made of the man and his importance to mathematics. Other men mentioned who have contributed to the study of mathematics are as follows: (1) Gauss, page 185, (2) Descartes, page 231, and (3) De Moivre, pages 389-396, and others.

Book B. (2) Like Book A, Book B mentions some of the important men, but it does not give any further mention as to their importance to mathematics. The rating of (2) for this book was given because it did have a listing of fourteen men and their importance to mathematics. These listings of the men and their contributions to mathematics were given between each chapter.

7. Different eras of mathematical development.

Book A and Book B. (1) Each book indicates the dates that the men mentioned in the book lived, but no further information is given.

8. Important men, events, and countries for their contributions to mathematics.

Book A. (1) As indicated in number 6 under this item of rating, this book makes some mention of the historical development of mathematics as a result of certain men. The authors did mention the Greeks as a contributor to mathematics, but no further mention was made in regard to countries. Events are not mentioned.

Book B. (2) In comparison to Book A, this book does a considerably better task of presenting the historical contributions made by the men of mathematics, the countries in which they resided, and the era in which they lived. However, a greater and more detailed development of the historical approach to mathematics is needed. For the purposes of this school district, supplemental material will also be needed.

I. Individual differences provided for in the textbook.

1. Exercises are indicated for their difficulty.

Book A. (1) Most of the exercises are simply stated and no indication is given for their difficulty. Several exercises have the difficulty of the problems noted. Exercises have enough material presented so as to develop an adequate understanding. Book A uses a (D) diamond notation for the more difficult problem exercises.

Book B. (3) Ample exercises given and the exercises are indicated by an alphabetical listing of A, B, or C for their difficulty for that specific topic. Little supplementary material would be

required for the problem exercises. In comparison to Book A, Book B had a greater range of problem exercises, better notation, and a greater number of exercises.

2. Content indicates problems that are unsolvable or a valid proof has not been derived. (These problems are offered as challenges to the student.)

Book A and Book B. (0) No problems, proofs, or other items were indicated in the table of contents, in the context, or elsewhere.

J. Content presented as a continuous pattern of mathematical thinking.

1. Topics presented in a logical sequence.

Book A and Book B. (2) The topics are presented in a very logical sequence; however, the topic of the straight line could be placed with the topics involving the conics, and no harm would be encountered in the presentation of the material to the student.

2. Topics presented as being workable in some areas but unworkable in other areas.

Book A. (1) In the development of complex numbers, page 378, is made of the various classifications of our number system and what equations are solvable in the set of integers, rational numbers, irrational numbers, and the complex numbers. These equations and their solution sets motivated the development for the complex numbers.

Book B. (2) The chapter on ordered fields, (Chapter II), presents a detailed development of the various properties of the integers and rational numbers and what properties of our number system are usable within each area of the number system. The field of complex numbers (Chapter 7), is developed as a result of a need for a larger number system. Chapter 7 indicates that the existing number system is inadequate. As a result, the complex numbers are developed as an extension of the existing number system.

K. Content relates to the real world where possible.

Book A. (2) A rating of two was given because the book devotes a considerable portion of the chapter on Boolean Algebra (Chapter 5) to an application of its content to computers and electrical circuits. Chapter 10, Probability, makes some references to the real world in its application of probability to genetics, public health, life insurance and other areas. Problem exercises, in the chapter on probability, relate to the real world with some practical applications indicated. Chapter 18, exponential and logarithmic functions, indicates an application for its content to chemistry, biology, and nuclear energy. Chapter 20, Limits, indicates an application for its content to the various branches of sciences. However, many chapters and problem exercises are given with no applications to the real world.

Book B. (2) Problem exercises, chapter introductions, and sample exercises are given that have some practical applications.

The chapters that involve sets, series, vectors, limits, circular and trigonometric functions, and probability all indicate some application to the real world. However, some chapters reveal no practical applications to the real world.

L. Content organized to benefit the student.

1. Material is presented for the student.

Book A and Book B. (3) For the most part, the content is presented in an organized pattern for the benefit of the student. The problem exercises could, perhaps, have some practical applications indicated in each chapter and each problem set of exercises.

2. Spiral approach used for redevelopment of previous terms in a new situation.

Book A. (2) The trigonometric ratios are developed in terms of x-y axis on pages 347-351. The analytic geometry relates the conics and their development to the x-y axis on pages 412-430. Other topics developed by the spiral approach include vectors, binomial expansion, probability, and others.

Book B. (2) Like Book A, has most of the topics related to analytic geometry and trigonometry introduced by the use of the x-y axis. Matrices and their transformations by the use of the trigonometric ratios are indicated on pages 547-549. Polar coordinates and the x-y axis are related as indicated on pages 485-489. Other topics related are sequences and mathematical induction, probability and the binomial expansion, and others.

D. METHODS

A. Presentation of the textbook material.

Book A. (1) Very little communication with the student is indicated. The definitions, the proofs, and the explanations of the sample exercises are merely presented in complete detail. The student merely reads the textbook and is expected to grasp the understanding of the presentation as given in the textbook.

Book B. (2) The explanations are such that they are talking to the students in that they ask questions such as "Why?" and "Do you see why something is as it is?". There are also other leading questions which ask questions which will enable the student to draw his own conclusions and to draw his own generalizations. The problem exercises are not designed or developed so as to communicate with the student as readily as are the sample exercises.

B. Discovery and axiomatic approaches used for the presentation of the material in the textbook.

1. Discovery approach.

Book A. (0) There was no indication of the discovery approach being used as a technique of presenting the material to the student. The explanation of the material was simply stated in complete detail with no leading questions or leading statements indicated for the students to make some generalizations on their own in regard to the material being presented.

Book B. (3) In comparison to Book A, Book B used the discovery approach as a process of presenting the material to the student. The explanations of the terms and the new material almost always asked questions of the students. The deletion of a question from any given explanation of the material was the exception.

2. Axiomatic approach.

Book A. (3) Book A develops the axiomatic approach in Chapter I and then uses the axiomatic approach throughout the book. In each chapter terms are carefully defined and stated, new axioms are introduced when needed, and the theorems are presented and proven in detail in regard to the previous material presented. The axiomatic approach was carefully developed on page 1 where the authors indicate that all terms cannot be defined. They also indicate the language to be used in regard to the presentation of the material in the book. Chapters 1 to 5 develop and use the axiomatic approach very carefully.

Book B. (3) The axiomatic approach is developed in detail in the first forty pages of the book. Chapter 1 deals with the language to be used, the defined and undefined terms to be used, and it develops the pattern of thinking and the presentation of the material to be used in the book. Pages 34, 35, and 36 give the axioms for equality, addition, multiplication, and distribution. Chapter 4 develops the axiomatic approach for the algebra of vectors in detail on pages 134, 143, and 151. The remaining chapters develop special properties, terms, and axioms where needed for the necessary presentations of the material.

C. New material presented proceeds from the concrete to semi-concrete to the abstract.

Book A and Book B. (2) In most cases, the material is presented in a pattern that the student is able to comprehend. Book A requires the assigning of certain selected exercises, page 389, in order to establish the desired pattern. Book B, page 238, presents the abstract first; then it proceeds to give several concrete examples. However, this is the exception rather than the general procedure followed in Book B as well as in Book A.

D. Review of the important material for each chapter.

1. Reviews.

Book A. (1) The book attempts to provide a review of the important material for each chapter by placing a chapter quiz or chapter test at the end of each chapter. However, Chapter 4 has no such review of the important material. Chapter 7 has a listing of miscellaneous exercises only. Many chapters end with problem exercises and no reviews or tests indicated as a means of reviewing the important topics in the chapter.

Book B. (3) Every chapter has a listing of the important topics to be remembered, and these topics are given in the Chapter Summary.

2. Tests exist for each chapter and for periodic cumulative reviews.

Book A. (1) Chapter tests exist for only a few chapters, and no cumulative reviews are present.

Book B. (2) Every chapter has a test, but no cumulative reviews are present in this book. However, in comparison to Book A, the chapter tests were a welcome method of review for the student.

E. Problem Exercises.

Book A. (2) A sufficient number of exercises exist; however, the book has a tendency to present too much material at any one time before it has any exercises given for developing an understanding of the material.

Book B. (3) In comparison to Book A, this book presents the material and follows with ample exercises. Usually, Book B presented a given amount of material and then had the exercises.

E. STUDENT AND TEACHER AIDS

A. Textbook is usable as a source of reference for the student and teacher.

Book A and Book B. (3) Both books are self explanatory and would make excellent reference books.

B. Separate editions exist for the teacher and student.

Book A. (0) Both use the same textbook, and no additional references exist for either the teacher or the student.

Book B. (3) Separate editions exist for the student and teacher.

C. Student's edition contains:

(a) Chapter review exercises, sample tests, and oral exercises.

Book A. (1) Chapters 1 and 2 contain thought questions for each chapter. Chapters 7 and 10 contain miscellaneous exercises at the end of each chapter. No review exercises, chapter summaries, or listings of important things to remember are given in the book.

Book B. (2) No review exercises are given. However, in comparison to Book A, a rating of 2 was felt justified in that a chapter summary and a listing of supplemental reading was made available in each chapter for the student.

(b) Sample tests.

Book A. (1) Chapter tests or quizzes are given for Chapters 1, 3, 5, 6, 9, and 10. The remaining chapters end their chapters with exercises or miscellaneous exercises.

Book B. (3) Chapter tests are given for each chapter. The chapter tests include short answer questions, word problems, proofs, and listing of the possible answers for a given question.

(c) Oral exercises exist as an aid for understanding for the student.

Book A and Book B. (0) Neither book had oral exercises indicated in the context. If the teacher desires to have oral exercises from these two books, he would have to investigate some of the problem exercises and then use these problems as possible oral exercises.

2. References made from previous topics to present continuity in the content.

Book A and Book B. (2) Most of the topics and the content are presented in a pattern that indicates continuity. Book A presented the chapter on the straight line separate and isolated from the other chapters on analytical geometry. Book B developed the same pattern as Book A in regard to the straight line and the sections on the conics. Students are asked to make reference to the x-y axis.

3. Selected problem exercises have the answers available in the student's textbook.

Book A. (0) No answers are given in the student's edition.

Book B. (3) The answers to the odd-numbered exercises are in the student's textbook. The proofs are not given for any of the problems in the student's edition.

D. Teacher's edition contains:

1. Teaching suggestions.

Book A. (0) None are present.

Book B. (3) Each chapter has suggested teaching aids as well as helpful hints for stressing important facts to be discussed with the students. These suggestions are presented in one textbook which is an aid for the teacher.

2. Enrichment materials.

Book A. (0) None were indicated in the book for the teacher to be used as an aid for presenting the material.

Book B. (2) The teacher's edition gave suggestions for finding enrichment material for most of the chapters. Some of the

materials suggested were paperbacks, annotations within the textbook, and the chapter reading tests.

3. Detailed solution key for the problem exercises.

Book A and Book B. (3) Each book contains a detailed solution key. Problems are worked in detail whenever such detail was required for the said problem.

E. Special features of the teacher's edition.

1. Annotated teacher's edition.

Book A. (0) No edition available or found for investigation.

Book B. (3) The book has special items of importance indicated and gives suggestions for teaching these items. An example of this is found on page 204 in the discussion of the slope of a line. Other numerous examples exist throughout the teacher's edition. In addition, the teacher's edition gives pages of references for a greater detailed explanation of the important items if they are needed for the teacher or the student.

2. Tests exist.

Book A. (1) Some tests were given for the individual chapters, but many chapters had no tests given as an aid for instruction.

Book B. (3) Every chapter had a test. In addition, there was a booklet available that contained suggested test questions other than those present in the textbook.

3. Suggested time table for the course.

Book A. (1) None was given. The book did indicate a possible sequence of topics that could be offered in a year and those that could be deleted from the course.

Book B. (3) The teacher's edition contains a chart (page 6) that indicates each chapter and the suggested number of days for that chapter for a minimum, average, or maximum course. The chart includes what sections can be deleted if the teacher desires to omit trigonometry, some analytic geometry, or any other topic.

F. SPECIAL FEATURES

The special features in existence in the textbook include:

A. Table of contents.

Book A and Book B. (2) Both books have a table of contents, but it is not as detailed as would be desirable for this course. Book B has a listing of the items for each chapter but no page reference. Book A has the items and page numbers given; its contents are in paragraph form, which makes the reading and selecting of important items rather difficult.

B. Variation in print.

Book A. (1) The only variation in the print is that the important items to be remembered or understood are in italics, and some important terms are in both quotes and italics. The observations for this variation are somewhat difficult to observe if one does not read closely or is not mentally alert when the observations are being made.

Book B. (3) Important solutions are in red print, theorems are indicated by a solid red rectangular block, important words are placed in dark black print, and important diagrams have black and red print to indicate the important items to observe in the given illustration.

C. Special mathematical tables.

Book A. (1) It contains the tables for the logarithms of factorials. No other tables are given. The book does not have a need for any other tables in either its explanations or problem exercises. However, the students always refer to a mathematics textbook when they need information from special mathematics tables.

Book B. (2) It contains tables for circular functions for the six trigonometric functions, a table for the six trigonometric functions, a table of squares and square roots, a table for the common logarithms, and a table for exponential functions. Cubes and cube roots and a statistical table of values for the standard deviation were not present. However, in comparison to Book A this book had more tables available for use for the student.

D. A special listing of the important formulas used in the textbook.

Book A and Book B. (0) Neither book had a special page established for the listing of the formulas used in the textbook.

E. Other features.

Book A. (1) On pages 639-641 a summary of all the symbols used in the textbook is given in an easy-to-read listing. No chapter

summary existed for any given chapter in this book.

Book B. (3) On pages 650-651 there is a listing of all the symbols used in the textbook. In addition, a listing of the Greek Alphabet is indicated, and the Script Alphabet is noted. A special chapter summary of the important items discussed in each chapter was given.

Frequent pictorial graphs are given as an aid for understanding the material presented in the section under discussion.

G. COMMENTS

Book A. (2) This book has been used as a source of reference for other mathematics courses taught, but this writer has not taught a large amount of material from the book. Very little opportunity exists for exploratory topics for the student.

Book B. Book B has been used as a source of reference for other mathematics courses taught, but this writer has not taught a large amount of material from this book. Exploratory material was not mentioned per se, but the book presented a brief bibliography of fourteen men. Since these men were to be part of the unit on the History of Mathematics, their bibliographies would be discussed and assigned for exploratory reading for the student.

The detailed evaluation for each book was indicated on the following outline as an aid to better illustrate the selection procedure for a senior mathematics textbook. Letter A was used to indicate the

rating for Book A and letter B was used to indicate the rating for Book B. In instances where the two books have identical ratings, letter A was given first and then letter B.

The outline for the selection of a senior mathematics textbook is as follows:

TABLE II
RATING SCALE FOR SENIOR MATHEMATICS TEXTBOOKS

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>I. Aims: (6 points)</u>			
<u>A. Aims are stated.</u>			
The aims are stated and are in complete agreement with those of the school district.	The aims are stated, but are not in complete agreement	The aims are stated but are for the most part in disagreement.	The aims are not stated.
(B)	()	(A)	()
<u>B. The aims are recognizable in the content.</u>			
The aims are clearly recognizable and in complete agreement with those of the school district.	The aims are recognizable but not in complete agreement.	The aims are not clearly stated and are in disagreement.	The aims are not recognizable.
()	(A) (B)	()	()
<u>II. Authorship and Research: (15 points)</u>			
<u>A. Classroom experience.</u>			
Teaching experience in mathematics at the high school level.	Teaching experience in junior high school or at the collegiate level.	Teaching experience at a level other than the secondary level.	No teaching experience.
()	(B)	(A)	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>II. Authorship and Research: (Continued)</u>			
<u>B. Educational training.</u>			
Major training in the area of the content of the book. (B)	Some training in the area of the content of the textbook. ()	Educational training only. ()	No training. (A)
<u>C. Content tested in the classroom.</u>			
Content of the book has been successfully tested by the author and his colleagues. ()	Content has been used as experimental units only by the author and his colleagues. (A)	Content has been based upon experience and observation, but not successfully tried in the classroom. (B)	Content has been written without regard to classroom use or classroom observations. ()
<u>D. Content conforms to various national curriculum committees.</u>			
Content in complete agreement. (B)	Most topics are in agreement. ()	Some topics included from the national committees. (A)	No topics included from the national committees. ()
<u>E. Authorship.</u>			
Has written several acceptable articles and books in mathematics. (B)	Has written some articles and a textbook. Their content acceptable. ()	Has written articles only. ()	Has no experience in writing. (A)

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (81 points)			
A. <u>Readable for the grade level intended.</u>			
Material is presented as being readily readable and understood in the given definitions, explanations, and problem exercises.	Content is readable in most areas.	Content is adequate, but certain areas cause difficulty in understanding the material.	Not readable for the grade level intended.
(B)	()	(A)	()
B. <u>Vocabulary.</u>			
Terms which are introduced are of significance. Terms are introduced and are used.	Terms introduced but not always used. Some terms are defined but are insignificant.	Some terms introduced but not defined, used, or are of any significance.	Nothing defined or explained.
(B)	()	(A)	()
C. <u>Content agrees with the district's proposed mathematics curriculum.</u>			
Complete agreement with the school districts proposed curriculum.	Several items not included but the majority of the items agree with the school district's recommended course.	Many items are not included in the textbook.	None of the items are in agreement with those of the school district's course outline.
()	(B)	(A)	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
D. <u>Proposed content in agreement with national committees recommendations.</u>			
Complete agreement.	Most items in the textbook agree with national recommendations.	Several items in agreement, others partially covered, and the rest not recommended for the given level of instruction.	No items in agreement.
(B)	(A)	()	()
E. <u>Accuracy of the content.</u>			
Accuracy is present in the definitions, explanations, and exercises.	Accuracy is present in most of the items presented.	Some inaccuracies are present in the explanations, definitions, and problem exercises.	Many inaccuracies are present in the content of the book.
(A)			
(B)	()	()	()
F. <u>Up-to-date content.</u>			
Content is up-to-date and uses up-to-date procedures for presentation of the material.	Most items are up-to-date. Several dated topics or articles included.	Contains items that are recommended for omission. Several items acceptable.	Content is dated and has been recommended for omission.
(A)	(B)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
G. Mathematical rigor emphasized in the following:			
1. <u>Proofs.</u>			
The development of proofs follow a logical argument and is there justification for the processes? Are the reasons indicated and the proofs demonstrated appropriate for the level of maturity of the student?	Proofs given but some items are deleted in the logical process of the proof.	Proofs are given but many proofs are left to be either proven by the student or simply accepted as they are presented.	The theorems are given, but the author indicates that the proofs are trivial and obvious. No proofs revealed.
(A)			
(B)	()	()	()
2. <u>Precision of the language.</u>			
Precise language used in the definitions, explanations, and problem exercises. Precise language to be used by the students in their responses and in their proofs.	Precision expressed in most areas of the text. Content contains some precision.	Precision of the language not stressed but is evident in the book.	No precision of the language expressed.
(A)	()	()	()
(B)			
3. <u>Foundations have been made for direct proofs before they are introduced.</u>			
Foundations are laid and clearly indicated so that the development of a proof is easily made.	Foundations are laid and indicated by the teacher. Development of a proof is made with assistance from the teacher.	Foundations made for some proofs and ignored by others. Proofs need some explanation by the teacher for greater understanding for the pupil.	Foundations are almost non-existent. Proofs must be developed by the teacher since the content does not reveal what is to be done or proven.
(A)			
(B)	()	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
4. <u>Foundations have been made for indirect proofs before they are introduced.</u>			
Indirect proofs have been introduced carefully. Foundations for their introduction have been made in detail.	Indirect proofs introduced, explained, and illustrated in most cases.	Indirect proofs introduced but require some time for their development for understanding for the student.	Indirect proofs mentioned only. If to be introduced the teacher must develop the entire process of indirect proofs.
(A)	()	(B)	()
5. <u>Are alternate proofs indicated for the proofs?</u>			
Alternate proofs are mentioned and some of the more important alternate proofs are given in detail for the student.	Some alternate proofs are indicated or mentioned. Several alternate proofs are given.	Alternate proofs mentioned. The details of the alternate proofs are indicated for only several proofs.	Alternate proofs mentioned or simply not given.
()	(A)	()	(B)
6. <u>Does the context for the proof clearly indicate where a proof or its demonstration starts and ends?</u>			
Clearly indicated for every proof.	Usually indicated.	Several careful readings of the proof are needed to determine the completion of the proof.	Not indicated.
(B)	(A)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>III. Content and Organization: (Continued)</u>			
H. <u>New learning theories and new approaches to problem solving are indicated in the content.</u>			
1. <u>Understanding stressed.</u>			
Content emphasizes understanding as well as computational skill.	Understanding emphasized but lacking in some areas.	Computational skill stressed. Some understanding stressed in some areas.	Understanding is forsaken. Computational ability stressed only.
(A)			
(B)	()	()	()
2. <u>Discovery approach indicated.</u>			
Content presented with the discovery approach being used as means of understanding.	Most items introduced use the discovery approach.	Many items are simply stated. Little discovery approach used.	Concepts and definitions are simply stated.
	(A)		
()	(B)	()	()
3. <u>When certain facts are established for two space, are the students encouraged to consider whether these properties are applicable to three space?</u>			
Every major area, where such generalizations are possible, encourages the student to transfer the applications to three space.	Generalizations are made for most areas and the students are encouraged to make the generalizations for three space.	Occasionally, generalizations are mentioned and students encouraged to apply it to three space.	Generalizations are made only. Or the generalizations developed only once and then never repeated.
	(A)		
()	(B)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>III. Content and Organization: (Continued)</u>			
<u>4. Development of the basic principles for the course.</u>			
Mathematical rigor is stressed throughout the book. Proofs, definitions, and problem exercises indicate mathematical rigor.	Rigor stressed in definitions and some proofs. Problem exercises usually stress the rigor.	Rigor stressed in some of the first few definitions and proofs. Later proofs ignore the desired rigor.	No mathematical rigor stressed.
(A)			
(B)	()	()	()
<u>5. Development of definitions in terms of the new approaches used in mathematics.</u>			
The definitions are developed by the use of examples and generalizations. Definitions are amply defined and stated.	Most definitions are clearly and accurately stated. Several definitions are simply stated and no examples given for the use.	Definitions are clear, but seldom used. Some inaccuracies exist in the definition itself or it is not used correctly. Definitions are usually stated and not developed by examples or problems.	Definitions are simply stated. No development used for the definition given.
(A)			
(B)	()	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
6. Does the author emphasize various areas of historical background for the major topics in mathematics?			
Historical development stressed for the major areas. History used as an aid for understanding the mathematics. ()	Historical development mentioned only. (B)	Several historical items mentioned or indicated. (A)	No mention is made of the historical development of the topics. ()
7. Does the author indicate the various eras of mathematical development?			
Ancient, medieval, middle, and modern ages indicated when possible. ()	Reference made to one or two eras only when possible. ()	Reference made only to an era when possible. (A) (B)	No mention is made. ()
8. Does the author indicate the important men, developments, and countries in the development of mathematical concepts and ideas?			
All items mentioned where possible. ()	Several items mentioned. Some references deleted. (B)	Mentioned several times only. (A)	No mention is made. ()
I. Individual differences provided for in the textbook.			
1. Exercises are indicated for their difficulty.			
Each exercise has the difficulty of the problems indicated. A sufficient number of exercises exist to develop an understanding of the material. (B)	Individuals differences indicated but some problems are of questionable difficulty. Some exercises need supplemental material to account for individual differences. ()	Individual differences indicated but many exercises require supplemental material. (A)	No provisions for individual differences. ()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
2. <u>Content indicates problems that are unsolvable or a valid proof has not been derived. (These problems are offered as challenges to the student.)</u>			
Problems from various areas of mathematics are offered as challenges to the student. Many problems and proposed solutions are offered the students.	One or two problems are mentioned. Some areas have several problems indicated.	One or two problems indicated. No possible solutions indicated.	None are indicated.
()	()	()	(A) (B)
J. <u>Content organization.</u>			
1. <u>Content presented in a logical sequence.</u>			
Content in a logical pattern. Content within a topic area, logically organized.	Content must be rearranged somewhat.	Considerable rearrangement necessary to follow a logical pattern.	No logical pattern present. Topics are completely isolated units of instruction.
()	(A) (B)	()	()
2. <u>Are certain problems indicated as being workable in one area but not workable in another area?</u>			
Example: $x^4 - 25$ factorable over:			
Integers. $(x^2 - 5)(x^2 + 5)$	Real numbers. $(x - \sqrt{5})(x + \sqrt{5})(x^2 + 5)$	Complex numbers. $(x - \sqrt{5})(x + \sqrt{5})(x - i\sqrt{5})(x + i\sqrt{5})$	
Clearly indicated and stressed in most cases. Questions are asked of the students in regard to possible restrictions on the operations of certain problems.	Indicated and illustrated. Questions occasionally asked of the student.	Indicated and questions asked of the students on occasion.	No mention is indicated for any restrictions.
()	(B)	(A)	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
III. Content and Organization: (Continued)			
K. <u>Associations made with the real world.</u>			
Each individual unit makes some association to the real world. References made to science, business, or other areas.	Some items but related, but most have some association to the real world.	Few items are related to the real world. Topics are simply presented with no references made as to their usefulness.	No associations presented as an aid to the usefulness of mathematics.
()	(A) (B)	()	()
L. <u>Content beneficial to the student.</u>			
1. <u>Material is presented for the student.</u>			
Material is presented to the student as material for the student and as material to be used by the student.	Most of the material is presented for the benefit of the student.	Some items are for the benefit of the student and other items are simply stated.	No benefits given or indicated.
(A) (B)	()	()	()
2. <u>Spiral approach used for redevelopment of previous terms in a new situation.</u>			
Frequent references made to previous content learned and how it applies to the new situation.	Some references made and how they apply to new situations.	References made only.	No references indicated.
()	(A) (B)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>IV. Method of Instruction: (21 points)</u>			
<u>A. Presentation of the textbook material.</u>			
The material communicates directly with the student. Textbook is a self-teaching unit.	Communication is apparent, however, some explanation is needed by the teacher.	Explanation of the material is needed for the understanding of the content. Teacher must explain a great deal of the material.	Much supplementary material needed for a detailed understanding.
()	(B)	(A)	()
<u>B. Discovery and axiomatic approaches used for the presentation of the material.</u>			
Discovery and axiomatic approaches used in the explanation, definitions, and the problem exercises.	The two processes are used in most of the material presented for consideration. Some explanations need extra explanations.	The two processes are used in some of the cases. Most cases, a great amount of explanation is needed for the presentation of the material.	The traditional approach is used in most phases of the material.
<u>1. Discovery approach.</u>			
(B)	()	()	(A)
<u>2. Axiomatic approach</u>			
(A)			
(B)	()	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>IV. Method of Instruction: (Continued)</u>			
<u>C. Abstract Material.</u>			
The abstract material is presented in a logical pattern for the student. Material proceeds from the concrete to the semi-concrete to the abstract material.	Material is usually presented in a logical manner.	Abstract ideas are simply stated. Little development is given for the abstract ideas.	Abstract material is stated and no development is given for the content.
()	(A) (B)	()	()
<u>D. Reviews and tests.</u>			
Periodic reviews and tests are given. Cumulative reviews and tests are indicated throughout the textbook.	Reviews and tests are usually given for each chapter and each teaching unit.	Chapter tests or chapter reviews given only, not both.	Neither chapter tests or chapter reviews given.
<u>1. Reviews.</u>			
(B)	()	(A)	()
<u>2. Tests.</u>			
()	(B)	(A)	()
<u>E. Problem exercises.</u>			
Ample exercises given with adequate instructions given for each problem set.	Ample exercises given, but some supplemental material is required for greater understanding of the content.	Exercises given, but supplementary material is needed for individual differences.	Few exercises given and individual differences within the exercises are void.
(B)	(A)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>V. Student and Teacher Aids: (39 points)</u>			
<u>A. References.</u>			
Adequate references are given for the student and teacher at the end of each chapter. (A) (B)	References are usually given for the teacher and the student. ()	References given but not in detail. References given only for some chapters. ()	No references given. ()
<u>B. Separate textbooks.</u>			
Teacher and student editions are separate. (B)	Same textbook. Same additional information given for the teacher's edition. ()	Same textbook. References available for the teacher. ()	Same textbooks. No additional references. (A)
<u>C. Student's edition.</u>			
<u>1. Review exercises and sample tests for each chapter are given.</u>			
<u>(a) Chapter review exercises.</u>			
Contains review exercises, oral exercises, chapter reviews and tests, and vocabulary listings for each chapter. ()	Contains most of these items, but not in great detail. (B)	Contains a chapter review or chapter tests only. (A)	No chapter tests or chapter reviews given. ()
<u>(b) Sample tests.</u>			
(B)	()	(A)	()
<u>(c) Oral exercises.</u>			
()	()	()	(A) (B)

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>V. Student and Teacher Aids: (Continued)</u>			
<u>2. References made from previous topics discussed to those that follow so as to develop a continuous pattern of mathematical thought.</u>			
Frequent references are made and the student is asked questions in regard to these references. Questions are asked of the student to recall previous material discussed.	References are made and the student is asked questions on occasion. (A) (B)	Some references are made, but not very frequently. ()	No mention is made of the previous topics discussed. ()
<u>3. Selected problems (odd or even numbered problems) have the answers given in the student's edition of the textbook.</u>			
Every problem set has answers for odd (or even) numbered exercises. (B)	Answers for only certain problems. ()	Answers for only word or else computational problems. ()	None (A)
<u>D. Teacher's edition.</u>			
<u>Suggestions for teaching the material are indicated for the teacher in the teacher's edition.</u>			
Identical to the student's textbook but contains specific suggestions for teaching each topic, suggestions for individual differences, suggestions for supplementary material and answers are in the textbook.	Contains some of these items but not in as great a detail.	Few suggestions given.	No additional information available other than the textbook used by the student.
<u>1. Teaching suggestions:</u>			
(B)	()	()	(A)
<u>2. Enrichment materials:</u>			
()	(B)	()	(A)

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
<u>V. Student and Teacher Aids: (Continued)</u>			
<u>2. The solution key contains a detailed solution for all the word problems and written exercises in the textbook.</u>			
Every problem is worked in detail.	Most of the problems are worked in detail.	Only the word problems are worked in detail.	Answers provided only for the problem exercises
(A)			
(B)	()	()	()
<u>E. Special features of the teacher's edition.</u>			
The teacher's edition is keyed to the student's textbook. Special chapter tests are available. A time table for teaching the content exists.	Special items are keyed only. Some answers are given. A few tests exist. A time table exists.	Several items are available.	No special features exist.
<u>1. Annotated teacher's edition.</u>			
(B)	()	()	(A)
<u>2. Tests exist.</u>			
(B)	()	(A)	()
<u>3. Suggest time table exists.</u>			
(B)	()	(A)	()
<u>VI. Special Features: (15 points)</u>			
<u>A. Table of Content.</u>			
Table of content given in detail with all items of importance given.	Table of contents given, some important items given.	Table of contents given.	No table of contents.
()	(A) (B)	()	()

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
VI. Special Features: (Continued)			
B. <u>Variation in print.</u>			
Important items are readily noticed by a different colored print or style of writing.	Most of the important items are indicated.	Some items are indicated with a different print or colored ink.	No variation exists.
(B)	()	(A)	()
C. <u>Special tables.</u>			
Table of squares and square roots, cube and cube roots, common logarithms, trigonometric functions, logarithms of the trigonometric functions, and statistical tables.	Most of the tables given. Several omitted.	Square and square root tables given. Logarithm tables given.	No tables given.
()	(B)	(A)	()
D. <u>Formulas listed:</u>			
A complete and detailed listing of all the formulas is given as used in the textbook.	Most of the formulas are given.	Basic formulas are given. The rest of the formulas are omitted.	No formulas are given in a separate listing.
()	()	()	(A) (B)

TABLE II (continued)

Excellent (3)	Good (2)	Fair (1)	Unacceptable (0)
VI. Special Features: (Continued)			
E. <u>Other features.</u>			
Style, readability, textbook size, study aids, and other items of educational importance.	Most of the items stated are given.	Few aids are given. Few special features.	No special aids and few features of importance are given.
(B)	()	(A)	()
VII. <u>Comments to be used for other areas of importance or consideration:</u>			
(8 points)			
<u>Book A.</u> (2) This book has been used as a source of reference many times for special topics and items that need a different approach in their explanations.			
<u>Book B.</u> (4) This book has been used as a source of reference for further explanations of some material, for sample exercises, and for problem exercises. The book provides variation in the problem exercises for individual differences. Exploratory material could be established if one investigates the bibliographies of the fourteen men given in the book.			

Below is the summarized results of the rating given for Book A and Book B. The topical headings are given with the subtopics listed below in the chart. Next to each subtopical heading is the assigned rating for the given book. The total score for each book is given at the bottom of each column.

I. AIMS	<u>Book A</u>	<u>Book B</u>
A. Aims specifically stated	1	3
B. Aims recognized from the content	2	2
II. AUTHORSHIP AND RESEARCH		
A. Classroom experience	1	2
B. Educational training	0	3
C. Content classroom tested	2	1
D. Author's content conforms to various national curriculum committees	1	3
E. Authorship (Previous articles)	0	3
III. CONTENT AND ORGANIZATION		
A. Readability	1	3
B. Vocabulary	1	3
C. Content in agreement with the districts curriculum	1	2
D. Content in agreement with the national committees recommendations	2	3
E. Accuracy of the content	3	3
F. Content up-to-date	3	2
G. Mathematical rigor		
1. Proofs	3	3
2. Precision of the language	3	3
3. Foundations made for direct proofs	3	3
4. Foundations made for indirect proofs	3	1
5. Alternate proofs given for some proofs	2	0
6. Development and completion of the proofs are indicated	<u>2</u>	<u>3</u>
Sub-total	34	46

	<u>Book A</u>	<u>Book B</u>
Sub-total	34	46
H. New learning theories and approaches		
1. Understanding and computation stressed	3	3
2. New approaches used in the problem exercises	2	2
3. Students encouraged to make generalizations	2	2
4. Development of basic principles	3	3
5. Definitions developed in terms of new approaches to mathematics	3	3
6. Historical approach used	1	2
7. Different eras of mathematical development	1	1
8. Men, events, and countries	1	2
Content allows for individual differences in the following:		
1. Exercises are indicated for their difficulty	1	3
2. Content contains unsolvable proofs and problems	0	0
J. Content presented as a continuous pattern of mathematical thinking		
1. Topics in a logical sequence	2	2
2. workable or unworkable in some areas	1	2
K. Content related to the real world	2	2
L. Content organized to benefit the student		
1. Material for the student	3	3
2. Spiral approach used	2	2
IV. METHODS		
A. Textbook communicates with the student in its presentation of the material	1	2
B. Discovery and axiomatic approaches		
1. Discovery approach	0	3
2. Axiomatic approach	3	3
Sub-total	65	86

	<u>Book A</u>	<u>Book B</u>
Sub-total	65	86
C. Material presented from concrete to the abstract	2	2
D. Review of the important material for each chapter		
1. Reviews	1	3
2. Chapter tests and periodic cumulative reviews	1	2
E. Problem exercises	2	3
V. STUDENT AND TEACHER AIDS		
A. Textbook usable as a source of reference	3	3
B. Separate editions	0	3
C. Student's edition		
1. Oral and review exercises and sample tests.		
(a) Chapter review exercises	1	2
(b) Sample tests	1	3
(c) Oral exercises	0	0
2. Mathematical continuity present in the content	2	2
3. Selected problems have the answers given in the student's text	0	3
D. Teacher's edition contains:		
1. Sources of information for		
(a) Teaching suggestions	0	3
(b) Enrichment materials	0	2
2. Detailed solution key for the problem exercises	3	3
E. Special features of the teacher's edition		
1. Annotated teacher's edition	0	3
2. Special tests available	1	3
3. Suggested time table and topics of importance	1	3
VI. SPECIAL FEATURES		
A. Table of contents	2	2
Sub-total	85	131

	<u>Book A</u>	<u>Book B</u>
Sub-total	85	131
B. Variation in the print	1	3
C. Special mathematical tables	1	2
D. Special listing of formulas	0	0
E. Other features	1	3
VII. COMMENTS		
Evaluator's comments	2	4
	—	—
Point totals	90	143

Book A has a total of 90 points while Book B has a total of 143 points over the seven main features and the 59 subtopics considered for evaluation of a senior mathematics textbook for the Davenport Public Schools. As a result of the evaluation and the total point values given to the two textbooks, it appears that Book B is the better book to be used for instruction in the Davenport Public Schools.

The rating of the two textbooks is only an indication of the writer's viewpoints as to how the two books compare. Several other members of the faculty would have to rate the two books using the established procedure to obtain a better over-all indication as to which textbook is the best for the curriculum established.

CHAPTER V

SUMMARY AND CONCLUSION

I. SUMMARY

This study was concerned with the following two important and related items as stated previously in Chapter I:

1. What content is recommended for the senior mathematics course?
2. What processes are needed for the selection of a senior mathematics textbook for the recommended course?

The study was for the Davenport Public Schools and the senior mathematics curriculum. In Chapter II a review of the past and present available literature revealed many proposed mathematics curriculums, but there were none that would adequately serve the needs of the given school district. In addition, the review of the current literature revealed some general guidelines for the selection of various mathematics textbooks. However, none of the reviewed guidelines indicated an adequate selection procedure for a senior mathematics textbook for the Davenport Public Schools. The selection procedure that was established was developed with the intentions that it would be of some advantage to the evaluators in their selection of a textbook for the senior mathematics course.

The first item involved in the report was to read the reports of the various mathematics curriculum committees to determine the

content of the senior mathematics course. In Chapter III, material from the various mathematics curriculum committees was gathered, investigated, deleted, and finally selected for the senior course. The curriculum reports were selected from the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, the Commission on Mathematics of the College Entrance Examination Board, the Ball State Teachers College Mathematics Program, and the National Council of Teachers of Mathematics. The material finally selected from these groups was used to develop the following senior mathematics curriculum:

1. The History of Mathematics (with written and oral reports.)
2. The Nature of Mathematics.
3. The Functions and Relations.
4. The Mathematical Induction and Sequences.
5. Vectors.
6. Probability and Statistics.
7. Matrix Algebra.

Guidelines for the selection of a senior mathematics textbook required an extensive review of the literature. Letters of inquiry were written to various publishing companies, to the Des Moines Public Schools, the Cedar Rapids Public Schools, the Allamakee School District, and to the National Council of Teachers of Mathematics. Score sheets, check lists, miscellaneous information acquired, and the results of the investigation of the literature were used for the eventual development of the outline for the selection of a senior

mathematics textbook. The outline that was developed for the selection procedure for a senior mathematics textbook contained the following major areas of importance:

1. Aims.
2. Authorship and Research.
3. Content and Organization.
4. Methods of Presentation.
5. Student and Teacher Aids.
6. Special Features.
7. Comments by the Evaluator.

The items deemed most essential to the selection of the senior mathematics textbook were the areas of content and organization, the methods of presentation, and the student and teacher aids within the textbook. Of these three items, the most important item would have to be the area of content and organization of the textbook.

The outline for the selection of a textbook was established with various questions being asked of the evaluator in regard to the textbook under evaluation. A rating of (3) for excellent, (2) for good, (1) for fair, and (0) for unacceptable was developed for the outline procedure of rating a textbook. If a rating of (0) was given, it was meant that the book did not have this item in its content. As a result of this rating of (0), the item would have to be developed by the evaluator for inclusion in the senior mathematics course. Both textbooks had several items that acquired a rating of (0). The rating

scale and the outline for textbook selection were given in Chapter III. The outline was developed with the needs of the course as an important item for consideration.

Chapter IV gave a detailed illustration of the selection procedure for two senior mathematics textbooks using the established outline and the proposed curriculum as a reference to be considered in the final selection of a senior mathematics textbook. The two textbooks considered for evaluation were Fundamentals For Advanced Mathematics, Book A, and Modern Introductory Analysis, Book B. The results of the evaluation of these two textbooks were given in detail in Chapter IV.

II. CONCLUSIONS

Chapter IV gave a detailed account of the rating for each textbook under consideration. The textbook that was selected as being the best suited for the curriculum of the Davenport Public Schools was Modern Introductory Analysis because of its higher numerical rating acquired from the established guidelines for the selection of a senior mathematics textbook. However, the textbook that was selected was not capable of meeting the entire demands of the established mathematics curriculum for the Davenport Public Schools. Supplementary material would still be needed for the topics on the History of Mathematics and Statistics.

In fact, it is doubtful if a mathematics textbook exists for the curriculum proposed! The reasons for the statement are that no textbook was found that had an adequate unit on history or the historical development of mathematics, a unit on statistics, an adequate development of matrix algebra, an adequate development for direct and indirect proofs, and the content presented in terms of the students' past and present needs.

Suggestions for use. The detailed demonstration of the outline in Chapter IV was given because it would serve as an excellent example for an examiner to use as a comparison in regard to the special features in a given textbook. The numerical rating given to a textbook would serve as an aid in comparison of the various textbooks under consideration if several individuals were rating the same textbooks.

This established outline was for the senior mathematics curriculum of Davenport; consequently, it would have to be studied carefully to be useful in its entire form for another school district. No two school districts are the same and their curriculums, likewise, are different. Thus, a slightly altered form for the selection of a textbook may be needed for a given school district. However, many of the items given in the outline would be beneficial for most senior mathematics curriculums and for the selection of a senior mathematics textbook. Some of the items considered as being of importance for the senior level course could be adapted to the

geometry and the advanced algebra courses. The items that could be considered as important for the two courses mentioned would be the stress on mathematical rigor and its development for proofs, the stress on the historical development of various ideas in mathematics, the stress of some application to the real world, and other items.

Limitations. The two textbooks were evaluated only on the established content of the outline. A better evaluation for the two textbooks would be possible if one actually taught the content from the two textbooks. The greatest test possible for a textbook is its use in the classroom and not an evaluation made from some established guidelines in an outline. Furthermore, the evaluation of the two textbooks was made by an individual and not by a group of educators. If a greater reliability is desired in a rating of a textbook, then the rating should be made by several individuals using the same outline procedures.

Recommendations. It is neither possible, nor desirable, that every state and local school system adopt a standard mathematics curriculum or a standard selection procedure for a textbook. Each locality has its own unique educational experiences that differ from the next school's educational experiences. However, the least that any school district can do in these times of rapid change is to continually review its current curriculums, its procedures, and its policies for textbook selection. The procedures used for today may differ

completely by the time tomorrow arrives. Yet, a general criteria should be developed and used every time a textbook, or a textbook series, is to be selected for a given school district.

Secondly, the senior mathematics curriculum and the selection procedures for an appropriate textbook for this curriculum are not the only areas of instruction that need serious consideration for evaluation and revision in the high schools of today. The general mathematics curriculum content needs to be reviewed and redeveloped. A selection procedure for a general mathematics textbook needs to be established and placed in operation as soon as possible.

A third recommendation is in regard to the training received by the teachers and those entering the profession. These professionals need to be given special training in regard to the problems of teaching mathematics (and other subject matter) to the average and below average students in the mathematics curriculum. The colleges and high schools must realize that there is a considerable number of students who will never walk through the hallowed halls of ivy in our colleges. The content that should be made available for this group of students needs careful consideration by all individuals in the educational process.

In the past, curriculums were altered every ten or more years. This time lag for curriculum development cannot exist if we as teachers, are to be of assistance to the student in his education. We must continually evaluate our courses that we are teaching, our methods of teaching, and the content that we are teaching if we are to keep

abreast of the ever-changing needs of society. Our schools' total curriculum, at all levels of instruction, must be continually up-dated and evaluated for maximum benefit to the student, the school, and the community.

Finally, the teachers must keep well informed of the current literature available in their respective teaching areas. They must keep abreast of the education within their areas and the educational practices that are being developed and used by the various national curriculum and experimental committees. In addition, too many teachers become quite complacent in their teaching because they are teaching only certain courses at a certain level of instruction. It is the responsibility of the teacher to know what is going on in his respective teaching areas. This means that the teacher should know what is being accomplished at the elementary level, the middle grade level, the junior high level, and the high school level of instruction in his respective teaching areas. The teachers at the high school level of instruction should also know what is being required at the collegiate level of instruction, in business, in industry, and in any other areas that would prove of benefit to the students.

If the revisions in the mathematics curriculum are made in detail and at all levels of instruction, then the mathematics curriculum will be ready for the teaching of the calculus as a two-semester course in the senior year of instruction. The two major items withholding this course from the high school at the present time are the availability of adequate teachers (this is a decreasing item of hinderance) and the reluctance of colleges to allow credit

for the calculus taught in the high schools. Adequate textbooks and supplementary materials exist for the presentation of the calculus to the high school student. In addition to the calculus course, there exists an even greater demand for the high schools to give instruction for a one-semester course in statistics. However, very few, if any, textbooks exist for the teaching of statistics in the high school.

The curriculum, as it exists today, may be adequate. By the time tomorrow arrives, it may be too late. Our objectives in education are to prepare the student for society and to enable the student to think for himself. Our curriculums must be established with this in mind or else education has failed!

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APPENDIX A

APPENDIX A

GUIDELINES FOR THE SENIOR MATHEMATICS RESEARCH PAPER

The paper can be written in conjunction with another course, however, you must obtain the approval of each instructor involved. The paper is required for the course and should be of sufficient length to cover the topic in detail. A paper of ten pages would be of minimum length. The paper will be due March 1, of this school year. The steps to follow for the paper include the following items:

- A. The Research Paper Defined.
The research paper is a presentation of facts which are:
 - 1. Based upon reading or consulting several specified sources.
 - 2. Presented according to a standard method of procedure.
 - 3. Limited to a relatively narrow phase of a subject.
 - 4. Original in relation to the selection, evaluation, expression, and conclusion.
- B. Bibliography.
Keep an accurate and complete record of each source of information that is consulted.
- C. Note Taking.
Use notes as the various sources are consulted for the research paper.
- D. Writing the Paper.
 - 1. Prepare an outline.
 - 2. Write the first draft of the research paper in your own words.
 - 3. Use quotations to emphasize an important point or as a proof for your conclusions.
 - 4. Write in a formal and objective style.
 - 5. Your imagination may be used in making deductions and conclusions.
 - 6. Reread and revise carefully.¹

¹Lorraine F. Dangle and Alice M. Haussman, Preparing the Research Paper (New York: College Entrance Publications, 1963), p. 1.

E. Direct Quotations.

1. A short quotation of fewer than five lines should be enclosed in quotations and placed in the context of the paper.
2. Long quotations of 5 or more lines should be placed in paragraph form, single spaced, extra indentation, no quotation marks, and leave double spacing between the context and the direct quote.

F. Footnotes.

1. Give credit to a direct quotation.
2. Give credit to an original or unusually interesting opinion or an interpretation which you have put in your own words.
3. Give credit for definitions, illustrations, and figures.

G. Bibliography.

1. Arrange the bibliography alphabetically with books first, periodicals second, and other sources third.

H. The Completed Paper.

1. Title page should include:
 - a. Title of the paper,
 - b. Writer's name,
 - c. Date submitted, and
 - d. Class.
2. Table of contents should reveal
 - a. The main divisions of the paper.
 - b. The page number on which the treatment of each main topic begins.
3. The pages of the text should be
 - a. Typed on one side only
 - b. Double spaced.
 - c. Documented with footnotes (single spaced)
 - d. Each page numbered, and (The first page of the text being page 1.)
4. Bibliography lists only the books and articles used in the final writing of the paper.¹

I. The Library Classification Systems has the following listings:

1. Pure Science-500.
2. History-900.
3. Further Classifications are:
 - a. Science-Q.
 - b. Technology-T.²

¹Ibid., pp. 1-23.

²Roberta H. Markman and Marie L. Waddell, 10 Steps In Writing The Research Paper (Woodbury, New York: Barron's Educational Series, Inc., 1965), pp. 110-112.

APPENDIX B

THE SENIOR MATHEMATICS COURSE OF STUDY:

(Analytic Geometry is a prerequisite)- This course is designed to lay the groundwork for the calculus. The course work will include: a history of mathematics, the nature of mathematics, functions and relations, mathematical induction and sequences, vectors, probability and statistics, and matrix algebra. These topics are approached from the standpoint of understanding and using formal proof. In addition, the course will require a research paper such that the student will have to do some individual investigations of some topic in mathematics and present the results in a formal paper and in an oral report.

Course of Study

I. History of Mathematics

A. Numeral Systems.

1. Early counting and recording of numerals by the;
 - a. Egyptians.
 - i. Moscow Papyrus (1850 B.C.)
 - ii. Rhind Papyrus (1650 B.C.)
 - b. Stonehenge (Salisbury - England)
 - i. The sight of an observatory.
 - c. Babylonians
 - d. Hindu - Arabic Numeral System
2. Early computation in addition and multiplication.
 - a. Illustrate the Roman Numerals in the two operations of addition and multiplication.
 - b. Add, subtract, and multiply in the early Egyptian notation.
3. Selected problems.
 - a. Simple (+, -, and x) using the Egyptian and Roman notation for numerals.

B. Contributions made by the Early Civilizations.

1. Egyptians.
 - a. The area of the circle is $\frac{8}{9}$ of the square of the diameter.
 - b. Illustrate the Egyptian process of multiplication by the succession of doubling operations.
 - i. Use (20x33).
 - c. State Problem Number 79 of the Rhind Papyrus.
 - i. Estate;

Houses	7
Cats	49
Mice	343
Heads of wheat	2401
Hekat Measures	16807
Total	19607

- ii. Indicate that this problem illustrates the powers of 7 and their summation.
 - 2. Babylonians.
 - a. Algebraic contributions.
 - i. Simultaneous equations.
 - ii. Squares and cubes of integers.
 - iii. Biquadratic equations.
 - iv. Indicate that the Babylonians were indefatigable tablet makers, computers of great skill, and were strong in algebra and geometry.
 - b. Geometry
 - i. The circumference was 3 times the diameter and the area $(1/12)$ the square of the circumference of the circle.
 - 3. Greeks.
 - a. Geometry (Study the men associated with geometry.)
 - i. Thales
 - a. A circle is bisected by the diameter.
 - b. The base angles of an isosceles triangle.
 - c. The vertical angles formed by two intersecting lines are equal.
 - d. Other items.
 - ii. Pythagoras and the Pythagoreans.
 - a. Indicate the man and the group.
 - b. Their study of number properties.
 - c. Arithmetic (The theory of numbers).
 - d. Pythagorean Theorem - $c^2 = a^2 + b^2$
 - e. Pythagorean Triples.
 - b. Algebra
 - 4. Hindu - Arabic.
 - a. This could be covered under the numeral systems of Part A.
 - b. Indicate their method of multiplication.
 - i. (135×12)

	1	3	5	
1	6	2	0	2

- C. Selected topics of interest.
 - 1. Chronology of pi (π).
 - a. Early values of pi were taken to have a value of 3.
 - b. The first scientific attempt to compute pi seems to be that of Archimedes around 240 B.C. called the classical method.
 - i. Increase the number of sides of an inscribed regular polygon and a circumscribed regular polygon closer and closer to the circumference of the circle.

- c. Indicate other men who have tried to compute pi by various methods.
 - i. Infinite series. (James Gregory)
 - ii. Probability. (Comte de Buffon)
 - iii. The use of the computer.
- 2. The three problems of Antiquity.
 - a. Introduction of the problem and state the rules for the three problems.
 - i. The rules for the construction are: "With the straightedge we are permitted to draw a straight line of indefinite length through any two given distinct points; with the compasses we are permitted to draw a circle with any given point as center and passing through any given second point."¹
 - b. Duplication of a cube.
 - i. Introduce the history of its origin.
The problem: How one can double a given solid while keeping the same shape?
 - c. Trisection of an angle.
 - i. Introduce the problem and its history.
The problem: Trisect any given angle.
 - a. Trisect a specific angle. (90° angle)
 - b. Tomahawk process.
 - 1. Assign this process to the class.
 - d. Quadrature of the circle.
 - i. Introduce the problem and its history. The problem:
Construct a square equal in area to a given circle.
- 3. Pythagorean Mathematics.
 - a. Topics from arithmetic.
 - i. Perfect numbers. (12 are known.)
 - a. A number is perfect if it is the sum of its proper divisors.
Example: $6 = 1 + 2 + 3$
 - ii. Deficient numbers.
 - a. A number is deficient if it is greater than the sum of its proper divisors.
Example: $8 = 4 + 2 + 3$
 - iii. Abundant numbers.
 - a. A number is abundant if it is less than the sum of its proper divisors.
Example: $12 = 6 + 4 + 3 + 2 + 1$
 - b. Topics from Part B, section 3, under the topic Greek Geometry.

¹Eves, op. cit., p. 82.

- c. Pythagorean logical scandal.
 - i. The discovery of the irrationality of $\sqrt{2}$ upset the basic assumption that everything depends upon the whole numbers. The definition of proportion, by the Pythagoreans assumed any two magnitudes to be commensurable.¹
 - d. Other topics as selected by the instructor.
- D. Men of Mathematics.
 - 1. Introduce these men and state some of their main contributions to mathematics and give their approximate date. Make a strong suggestion that this is a good list from which to select a topic for their research paper. Many men have been deleted for reasons of time and space. Some of the men are as indicated:
 - a. Zeno 450 B.C., (Paradox of motion.)
 - b. Aristotle 340 B.C., (Deductive logic.)
 - c. Euclid 300 B.C., (Elements, perfect numbers.)
 - d. Archimedes 225 B.C., (One of the greatest mathematicians of antiquity, circle and sphere, and pi.)
 - e. Ptolemy 150 B.C., (Trigonometry.)
 - f. Fibonacci 1202 (Arithmetic, Fibonacci sequence, and others.)
 - g. Copernicus 1530 (Trigonometry and planetary theory.)
 - h. Ferrari 1545 (Quartic equations.)
 - i. Tartaglia 1545, (Cubic equations.)
 - j. Cardano 1545, (Algebra.)
 - k. Galileo 1600, (Falling bodies and projectiles.)
 - l. Kepler 1610, (Laws of planetary motion, and continuity.)
 - m. Napier 1614, (Logarithms.)
 - n. Fermat 1635, (Number theory and probability.)
 - o. Descartes 1637, (Analytic geometry.)
 - p. Pascal 1650, (Probability and the Pascal Triangle.)
 - q. Gregory 1670, (Pi, infinite series, and the binomial theorem.)
 - r. Newton 1680, (Gravitation, dynamics, fluids, and calculus.)
 - s. Leibniz 1682, (Calculus and determinants.)
 - t. DeMoivre 1720, (Actuarial mathematics, probability, and complex numbers.)
 - u. Gauss 1820, (Number theory and algebra.)
 - v. Others.²

¹Ibid., p. 63.

²Ibid., pp. 383-389.

- E. Contributions to the various branches of mathematics.
1. Algebra and Arithmetic.
 - a. Countries. (Arabs, Babylonians, and others.)
 - b. Men who have made some contributions to the area include:
 - i. Euclid, number theory.
 - ii. Eratosthenes, prime numbers.
 - iii. Fibonacci, number series and others.
 - iv. Oresme, fractional exponents.
 - v. Others.
 2. Geometry. (Plane, analytic, and others.)
 - a. Euclid (Elements).
 - b. Pythagoras and the Pythagoreans (Pythagorean Theorem).
 - c. Descartes (Analytic geometry).
 - d. Lambert (Parallel postulate).
 - e. Bolyai (Hyperbolic geometry).
 - f. Lobachevsky (Hyperbolic geometry).
 - g. Abel (Elliptic geometry).
 - h. Riemann (Elliptic geometry).
 - i. Others.
 3. Trigonometry.
 - a. Countries (Arabs, Greeks, Hindu, and others).
 - b. Men:
 - i. Hipparchus (Astronomy and a table of chords).
 - ii. Ptolemy (Modification of the table of chords).
 - iii. Beg (Trigonometric tables).
 - iv. Copernicus (Developed trigonometric tables).
 - v. Rheticus (Trigonometric functions).
 - vi. DeMoivre (Complex numbers).
 - vii. Bernoulli (Polar coordinates).
 - viii. Others.
 4. Probability.
 - a. Pascal and Fermat (These two helped lay the foundations of the mathematical theory of probability).
 - b. Bernoulli (Probability theory).
 - c. DeMoivre (Doctrine of Chance).
 5. Calculus (If student interest is present for this topic).
 - a. Newton and Leibniz are the two men credited to the forming of the calculus).
 - b. Taylor (Taylor expansion of a function).
 - c. Maclaurin (Worked on the Taylor series).
 - d. Lagrange (Attempted to make the previous notations in the calculus more rigorous and some work on partial differential equations).
- F. Word origins as they are encountered in the Senior Mathematics Course.
- G. Mathematical Conjectures.

1. Goldbach's Conjecture:
Every even integer, except 2, seemed representable as the sum of two primes.
2. The three problems of antiquity restated and briefly discussed.
3. Pierre de Fermat's Conjecture: (Found to be incorrect.)
 $f(n) = 2^{2^n} + 1$ is prime for all nonnegative integral n .
4. The problem of finding a function $f(n)$ which, for all positive integral n , will yield only prime numbers.
 - a. $f(n) = n^2 - n + 41$ works for all $n \leq 41$, but fails at $f(41) = 41^2$.
5. Koenigsburg Bridge Problem as developed by Euler.
6. Other conjectures as deemed essential by the instructor.¹

II. Nature of Mathematics.

A. Introduction to groups.

1. Define:
 - a. Ordered pair.
 - b. Binary operation.
 - c. Set of odd and even integers.
 - d. Closure property.
 - e. Mathematical systems.
2. Definition of a group:
 - a. A group is a non empty set of elements and a binary operation (\circ) for which the following conditions are satisfied:
 - i. Closed with respect to (\circ).
 - ii. The associative law holds with respect to (\circ).
 - iii. An identity element exists.
 - iv. There exists an inverse element.²
3. Commutative (Abelian) Groups.
 - a. A group is commutative or abelian if $a \circ b = b \circ a$ for all elements a and b in the group.³
 - b. Assign problems for both groups involving the operations of (+, -, x, and /). Use the operation of subtraction and division to illustrate non-commutativity.
4. Subgroups.
 - a. State the definition.
 - b. Give several examples of a subgroup:
 - i. Set of all even integers is a subgroup of all the integers under addition.
 - ii. (1) and (-1) under multiplication form a subgroup but fail for addition.
5. Group generators.
 - a. State the definition.
 - b. Give several examples.

¹Ibid., pp. 1-389.

²Dinkines, op. cit., p. 5.

³Ibid., p. 10.

6. Cyclic groups.
 - a. State the definition.
 - b. Give several examples.
 7. Theorems about groups.
 - a. In a group $G (o)$, if $y (o) a = a$, then $y = e$ where e is the item to be proven as the identity element.
 - b. In a group $G (o)$ the identity element is unique.
 - c. In a group $G (o)$ each element has a unique inverse.
 - d. And others.¹
- B. Introduction to rings and fields.
1. Definition of a ring:
 A ring R is a nonempty set of elements satisfying the following axioms under the operations of (o) and $(\#)$.
 - a. R is an abelian group under (o) .
 - b. R is closed with respect to $(\#)$.
 - c. The operation $(\#)$ is associative.
 - d. The distributive laws hold; that is,
 - i. $a (o) [b \# c] = a (o) b \# a (o) c$ and
 - ii. $a \# [b (o) c] = a \# b (o) a \# c$ when $a, b,$ and c are in R .²
 2. Theorems about rings.
 - a. $a)-b) = (-a)b = -(ab)$.
 - b. $(-a)(-b) = ab$.
 - c. $a(b - c) = ab - ac$.
 3. Integers (Modulo n).
 - a. Define (Modulo n).
 - b. Work problems involving modulo 3, 5, and 7.
 4. Assumptions concerning the real and complex numbers.
 - a. Define the following:
 - i. Infinite decimal.
 - ii. Repeating decimal.
 - iii. Terminating decimal.
 - iv. Real numbers.
 - v. Irrational numbers.
 - vi. Complex numbers.
 5. Definition of a field.
 - a. "A field is a commutative ring of at least two elements, in which there is a unit and every nonzero element has a multiplicative inverse."³
 - i. The integers fail because of no multiplicative inverse.

¹Ibid., pp. 29-34.

²Ibid., p. 58.

³Ibid., p. 77.

- ii. Rational numbers form a field.
- iii. Real numbers form a field.
- iv. Complex numbers form a field.
- 6. Theorems about fields.
 - a. A field has no divisors of zero.
 - b. Others as deemed essential by the instructor.¹
- 7. Ordered fields.
 - a. Field properties of equality are:
 - i. Reflexive.
 - ii. Symmetry.
 - iii. Transitive.
 - b. Properties of addition.
 - i. Closure.
 - ii. Associative.
 - iii. Identity Element.
 - iv. Additive Inverse.
 - v. Commutative.
 - c. Properties of multiplication.
 - i. Closure.
 - ii. Associative
 - iii. Identity Element.
 - iv. Multiplicative Inverse.
 - v. Commutative.
 - vi. Distributive Property.
 - vii. Substitution Property.
 - d. Order properties.
 - i. Comparison.
 - ii. Transitive
 - iii. Order of addition.
 - iv. Order of multiplication.
 - v. Substitution property.
 - e. Properties for proving theorems.
 - i. Cancellation law for addition.
 - ii. Cancellation law for multiplication.
 - iii. Multiplication property of zero.²
- C. Complex numbers.
 - 1. Definition of a complex number where it is written as an ordered pair $(a + bi)$ and the properties of a field for real numbers apply to this ordered pair $(a + bi)$.
 - 2. In the ordered pair $(a + bi)$, a is the real part and bi is the imaginary part.
 - 3. Graphical representation of complex numbers.

¹Ibid., pp. 71-84.

²Dolciani, Beckenbach, Donnelly, Jurgensen, and Wooten, op. cit., pp. 33-63.

- a. The real part, a , is the x axis and
- b. The imaginary part, bi , is the y axis.
- 4. The four operations on the complex numbers.
 - a. Addition.
 - b. Subtraction.
 - c. Multiplication.
 - d. Division.
- 5. Equality of complex numbers.
 - a. $a + bi = 4 + 7i$ where $a = 4$ and $bi = 7i$.
- 6. Complex numbers and quadratic equations.
 - a. Determine the nature of the roots of a quadratic equation by the use of the discriminant.
- 7. Polar form of a complex number.
 - a. $x = r \cos \theta$.
 - b. $y = r \sin \theta$.
 - c. r represents the radius vector.
- 8. DeMoivre's Theorem.
 - a. $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ where n is an integer.
- 9. Roots of a complex number.
 - a. Find the four roots of 16, i.e. $x^4 = 16$.
 - b. Find the 3 roots of 1, i.e. $x^3 = 1$.

III. Functions and Relations.

- A. Definitions:
 - 1. Domain and range.
 - 2. Relation - any set of ordered pairs.
 - 3. Functions - any pairing of the members of one set (the domain) with the members of another set (the range) so that each member of the domain has only one partner in the range.
- B. Types of functions. (An intensive and brief review only).
 - 1. Linear function $f(x,y) = ax + by + c$
 - a. when the function has the value 0 it is the straight line, $0 = ax + by + c$
 - b. $ax + by + c > 0$ or < 0
 - c. $ax + by + c \geq 0$ or ≤ 0
 - d. Solve and graph.
 - 2. Quadratic functions.
 - a. $[(x,y) : y = x^2]$ and more generally given by the quadratic polynomial, $ax^2 + bx + c$, where a , b , and c are real numbers and $a \neq 0$.
 - b. Solve and graph.

3. Polynomial functions of the form:
 - a. $[(x,y): y = 3x^2 + 5x + 1]$
 - b. Solve and graph.
 - i. Remainder and Factor Theorems.
4. Trigonometric functions of the (sin, cos, tan, sec, csc, and cot) angular measurement.
5. Circular functions of the (sin, cos, tan, sec, csc, and cot) radian measure.
6. Periodic functions.
 - a. Sin and cos periodic functions.
 - b. Graph these two functions from 0° to 360° .
7. Exponential functions.
 - a. $[(x,y): y = b^x, b > 0, b \neq 1]$
 - b. Graph. Examples: base 2, base 3, and base 10.
8. Logarithmic functions.
 - a. $[(x,y): y = \log_b x, b > 0, b \neq 1]$
 - b. Graph. Examples: base 2, base 3, and base 10.
9. Inverse functions.
 - a. Exponential and logarithmic functions.
 - b. A function f and its inverse f^{-1} or $[f^{-1}(x)] = x$
 - c. $[(x,y): y = \sin x]$ the inverse is $[(x,y): x = \sin y]$
 - d. Inverse relation $y = \sin^{-1} x$ or $y = \arcsin x$
 - i. $y = \cos^{-1} x$
 - ii. $y = \tan^{-1} x$
 - iii. $y = \cot^{-1} x$
 - iv. $y = \sec^{-1} x$
 - v. $y = \csc^{-1} x$
10. Other inverse functions as deemed essential by the instructor.

IV. Mathematical Induction and Sequences.

- A. Principle of Mathematical Induction:

"If a property is true of the number 1, and if we prove that it is true of the positive integer $k + 1$ provided it be true of the positive, integer k , it will be true for all positive integers."¹
- B. Proofs by mathematical induction.
 1. $1 + 3 + 5 + \dots + (2n - 1) = n^2$
 2. Given: $5 + 9 + 13 + 17 + \dots$
Find two formulas and verify they are equal by mathematical induction.
- C. Definition of a sequence and series.
 1. Sequence: A set of numbers that are in a particular order which is inherited from the natural numbers.
Example: (3, 5, 7, 9, . . .)

¹Herberg and Bristol, op. cit., p. 514.

2. Series: Indicates that the sum of the terms of sequence are to be found.
Example: $(3 + 5 + 7 + 9 + \dots)$
- D. Progressions: (also called a sequence).
1. Arithmetic. Any sequence in which each term after the first is found by adding a constant number, called the common difference, to the preceding term.¹
 - a. $l = a + (n - 1)d$ where l represents the last term, a the first term, d the common difference, and n the n th term.
 - b. $S_n = n/2(a + l)$ where S_n is the sum of the first n terms.
 2. Geometric. Any sequence in which each term after the first is the product of the preceding term and a constant number, called a ratio, is called a geometric sequence.²
 - a. $L = ar^{n-1}$ where a is the first term of the sequence, L is the last term, and n represents the n th term. The letter r represents the common ratio.
 - b. $S_n = \frac{a - ar^n}{1 - r}$, $r \neq 1$, and S_n represents the sum of the first n terms.
- E. Binomial Theorem.³
1. $(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + b^n$
- F. Infinite Sequence and Series.
1. An infinite sequence which has a limit is said to converge or to be convergent.⁴
 2. Nondecreasing sequence (monotonic increasing).
 3. Nonincreasing sequence (monotonic decreasing).
 4. Divergent sequence: a sequence that does not have a limit.
- G. Infinite Geometric Series.
1. The sum of an infinite geometric series when $|r| < 1$ is represented by the formula:
$$S = \frac{a}{1 - r}$$

¹Ibid., pp. 105-106.

²Ibid., p. 130.

³Ibid., p. 496.

⁴Dolciani, Beckenbach, Donnelly, Jurgensen, and Wooten, op. cit., pp. 95-112.

H. Axiom of completeness.

1. Definition: Every bounded, nondecreasing (or increasing) sequence of real numbers converges, and its limit is a real number.¹

V. Vectors.

A. Definition of a vector:

A vector is a quantity having magnitude and direction.

B. Properties of a vector.

1. Initial point.
2. Terminal point.
3. Standard position.
4. Resultant (sum of two vectors).
5. Components.

C. Vector addition, multiplication, and other properties.

1. Number line.
2. Cartesian plane.
3. Additive inverses.
4. Addition properties for:
 - a. Closure.
 - b. Substitution.
 - c. Commutative.
 - d. Associative.
 - e. Identity.
 - f. Inverse.
5. Vector subtraction.
6. Scalar multiplication.
 - a. Definition of a scalar.
 - b. Multiplication properties:
 - i. Closure.
 - ii. Commutative.
 - iii. Associative.
 - iv. Identity
 - v. Zero product.
 - vi. Negative one.
 - c. Other properties.
 - i. Substitution
 - ii. Distributive
 - iii. Norm of a vector.

D. Geometric representation of a vector.

¹Ibid., p. 108.

1. Prove some theorems using vectors.
 - a. The diagonals of a parallelogram bisect each other.
 - b. If two sides of a quadrilateral are equal and parallel, then the other two sides are also equal and parallel.
- E. Basis of a vector. (Optional for discussion.)
- F. Inner product of two vectors.
 1. Inner product (sometimes called the scalar or dot product).
- G. Vector properties.
 1. Symbolism
 2. Dot product; if a and b are two vectors with a common starting point, then $a \cdot b = |a||b| \cos \theta$ where θ is the angle between a and b and $0 \leq \theta \leq \pi$
 3. Perpendicularity.
 4. Other properties.
- H. Free vectors:
 Vectors that are parallel to one another and are free to be moved to coincide with another vector.
- I. Vector application to forces.
 1. Parallel forces.
 2. Perpendicular forces.
 - a. Perpendicular vectors.
 - b. Perpendicular components.
- J. Polar coordinates and graphs.
 1. Pole.
 2. Polar axis.
 3. Radius vector or polar distance.
 4. Polar coordinates.
 5. Polar distance formula.
 6. Polar sketching techniques.
- K. Polar coordinates and equations.
 1. Circles.
 - a. $r = 2$.
 - b. $r = b \sin \theta$.
 - c. $r = 2 \cos \theta$.
 2. Lines.
 - a. $r \sin \theta = b$.
 - b. $r \cos \theta = 2$.
 3. Special polar equations (cardioid, 4 leaf rose, and others).
 4. Relationships between polar and rectangular coordinates.

VI. Probability and Statistics.

A. Definition of:

1. Statistics.
2. Probability.

B. Statistics.

1. Define:

- a. Mean.
- b. Median.
- c. Mode.
- d. Quartile.
- e. Range.
- f. Skewness.

2. Random sampling.

3. Other sampling procedures.

4. Ungrouped data.

- a. Frequency diagram.
- b. Cumulative graph.

5. Grouped data.

1. Frequency histogram.
2. Cumulative polygon.
3. Frequency table.

6. Percentiles.

7. Measure of central tendency.

- a. Mean.
- b. Median.
- c. Mode.
- d. Other.

8. Standard deviation and its calculations.

- a. Grouped data.
- b. Compute the necessary totals.
- c. Find the standard deviation.

9. Normal probability curve and its application.

1. Discuss the use of the curve and its meaning completely.
2. Give numerous examples for finding ± 1 and ± 2 standard deviations for the curve.

C. Probability.

1. Introduction by the "intuitive approach".

2. Experiments with:

- a. Coins.
- b. Die and dice.
- c. Deck of cards.

3. Sample spaces.

- a. Defined as the elements that correspond in a one-to-one relationship to the outcomes of an experiment.
- b. Examples in detail with one and two rolls of a die and dice, respectively.

4. Events and their probabilities.
 - a. Coin(s).
 - b. Die and dice.
 - c. Cards.
 - d. Other.
5. Probabilities and sets.
 - a. Mutually exclusive (cannot both happen in a single event).
 - b. Independent probabilities.
 - i. First event does not depend upon the second.
 - c. Dependent probabilities (conditional)
 - i. Two events that are not independent are called dependent events.
 - a. What happens on the first event affects the outcome of the second event.
6. Compulsory events.
 - a. Events that must happen.
7. Random (drawings and numbers) and the use of the table of random numbers.
8. Permutations.
 - a. Defined as: You are concerned with the arrangement of the items that have been selected. Order is important.
 - b. Linear and circular permutations.
9. Combinations.
 - a. Defined as: You are concerned with the elements selected and not their arrangements or their order.
10. Binomial Distribution.
 - a. Binomial theorem discussed again.
 - b. Pascal's Triangle.
 - c. Coefficients table.
 - d. Definition for the Binomial Distribution.
 - i. The number of trials is fixed and
 - ii. The trials are each independent of the other trials that have been completed.

VII. Matrix Algebra.

- A. Definition of a matrix.
 1. A rectangular array of numbers is called a matrix.
 2. The notation to be used:
 - a. $\begin{bmatrix} \end{bmatrix}$
 - b. $\begin{pmatrix} \end{pmatrix}$
 - c. $\begin{Bmatrix} \end{Bmatrix}$
 3. Dimension (or order) - the number of rows indicate the order of the matrix.
 4. Row (vectors)

5. Column (vectors).
 6. Zero matrix.
 7. Identity matrix.
 - a. The zero matrix is the identity matrix.
 8. Transpose matrix. (A^T notation used.)
- B. Operations with matrices.
1. Equality.
 2. Addition.
 3. Multiplication.
 4. Multiplicative Inverse.
 5. Product of a scalar and a matrix:
- $$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \Rightarrow 3(A) = \begin{bmatrix} 3(2) & 3(-1) \\ 3(0) & 3(3) \end{bmatrix}$$
- C. Using matrices to solve problems.
1. Introduction to determinants.
 2. Triangulation method (diagonal method).
 - a. Evaluate determinants.
 3. Formulation in terms of matrices.
 - a. The ability to represent systems of linear equations in matrix form.
 - b. Matrix function.
 $f: X \rightarrow Y$ where both the domain and range are sets of matrices.
 4. Solution by means of matrices.
 - a. Solve linear equations in 2 and 3 unknowns.
 5. Matrix inversion.
 - a. Form the inverse matrix.
 6. Linear systems in general.
 - a. Same as item 4 in the above but in greater depth if so desired by the instructor.
- D. Linear transformations.
1. Brief introduction only.
 2. Geometric.
 3. Matrix transformation.
 4. One-to-one linear transformation.
 5. Rotation.
- E. Algebra of matrices.
1. Introduction.
 2. 2x2 matrices.
 - a. Inverse.
 - b. Ring of a matrix.
- F. Determinants again introduced.
1. Defined.
 2. Finding determinants.
 - a. Properties of a determinant.
 - b. A square matrix can be paired with its determinant.
 3. Using determinants to solve problems.
 4. Inverses.

G. Augmented matrix.

1. An augmented matrix is formed by the coefficient matrix and the column matrix.

Example: $3x + 2y = 5$

$7x + 4y = 9$

$$\left[\begin{array}{cc|c} 3 & 2 & 5 \\ 7 & 4 & 9 \end{array} \right]$$

H. Optional. Matrices to study the properties of:

1. Groups.
2. Rings.
3. Fields.

VIII. Logic and Method of Proof. (Optional)

A. Language and symbols.

1. Conjunction and negation.
2. Symbols.

B. Proofs.

1. Truth tables.
2. Syllogisms.
3. Indirect proof.
4. Law of detachment.

C. Sets applied to:

1. Algebra.
2. Logic.
3. Boolean Algebra.*

*The material for this outline was derived from a variety of sources. The main contributions were from the following:

1. Dolciani, Beckenbach, Donnelly, Jurgensen, and Wooten.
2. Herberg and Bristol.
3. Dinkines.
4. The material from SMSG, UICSM, CEEB, and Ball State.

APPENDIX C

APPENDIX C

TEXTBOOK EVALUATION FORM FOR _____

(Name of series) _____

TOPIC	EVALUATION COMMENT	Your evaluation score.
I. AIMS. (6 points)		
II. AUTHORSHIP AND RESEARCH. (15 points)		
III. CONTENT AND ORGANIZATION. (81 points)		
IV. METHOD OF INSTRUCTION. (21 points)		
V. STUDENT AND TEACHER AIDS. (39 points)		
VI. SPECIAL FEATURES. (15 points)		
VII. ADDITIONAL COMMENTS. (8 points)		
Maximum points possible. (185 points)		Your total score= _____